Superconductivity

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The BCS Gap

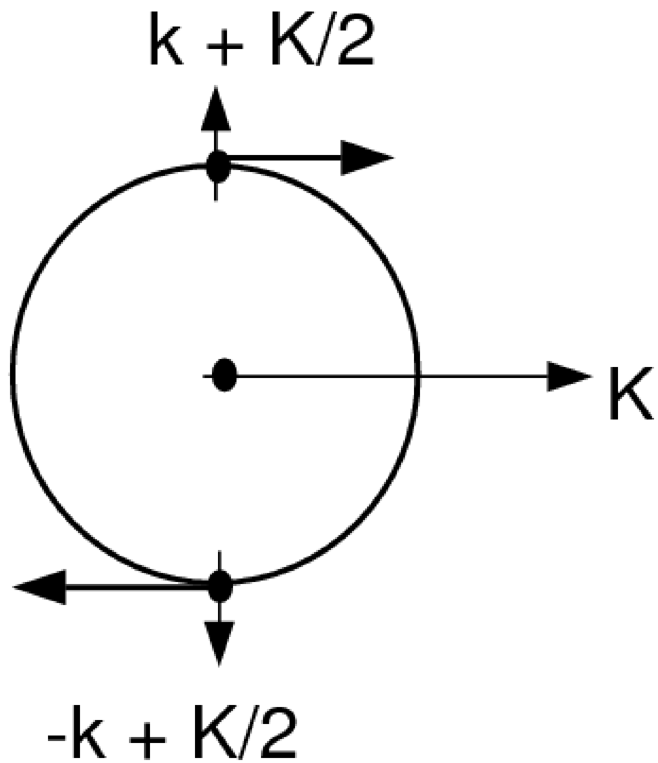
$$W_{BCS} = \sum_k 2\xi_k \frac{1}{2}\left(1 - \frac{\xi_k}{E_k}\right) - \frac{L^3\Delta^2}{V_0}$$

$$\downarrow \qquad \text{Lots of algebra (See I\&L)}$$

$$W_{BCS} = -\sum 2E_k v_k^4$$

Now recall that the probability that the Cooper state (k ↑, k ↓) was occupied, is given by $w_k = v_k^2$. Thus the first pair breaking excitation takes $v_{k'}^2=1$ to $v_{k'}^2 = 0$, for a change in energy

$$\Delta E = -\sum_{k \neq k'} 2v_k^4 E_k + \sum_k 2v_k^4 E_k = 2E_{k'} = 2\sqrt{\xi_{k'}^2 + \Delta^2}$$



$$k = \frac{1}{2} K$$

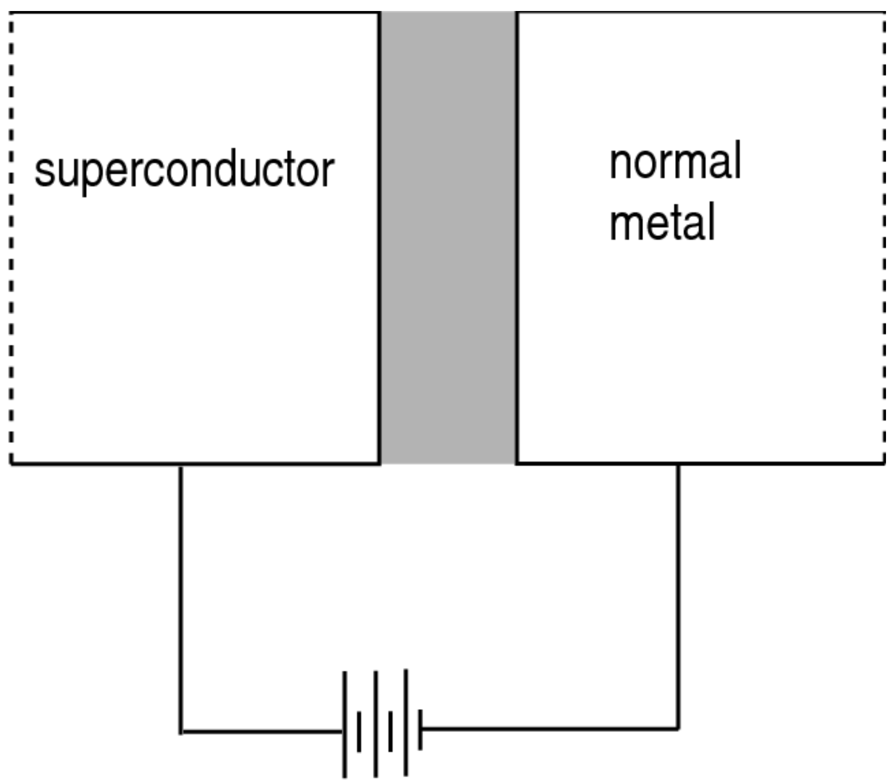
$$R = \frac{r_1 + r_2}{2}$$

Then since $\xi_{k'} = \frac{\hbar^2 k'^2}{2m} - E_F$, the smallest such excitation is just

$$\Delta E_{min} = 2\Delta$$

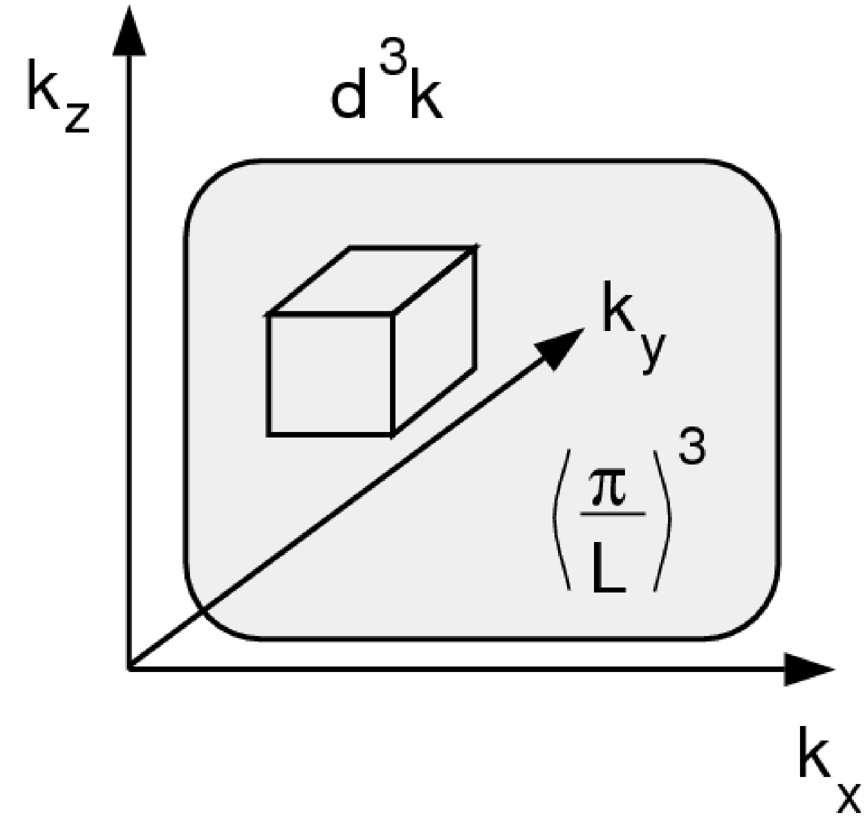
This is the minimum energy required to break a pair, or create an excitation in the BCS ground state. It is what is measured by the specific heat $C \sim e^{-\beta 2\Delta}$ for $T < T_c$.

Now consider some experiment which adds a single electron, or perhaps a few unpaired electrons, to a superconductor (ie tunneling).



Since it is a single electron, its energy will be

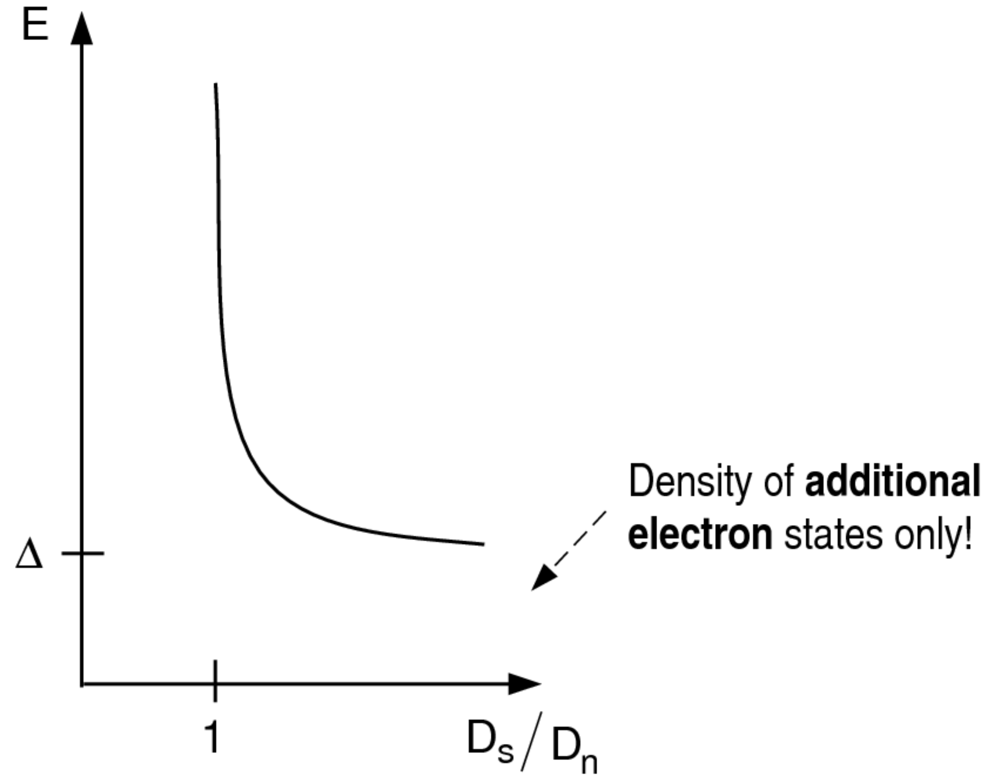
$$E_k = \sqrt{\xi_k^2 + \Delta^2}$$



$$D_s(E_k)dE_k = D_n(\xi_k)d\xi_k$$

In the vicinity of $\Delta \sim \xi_k, D_n(\xi_k) \approx D_n(E_F)$ since $|\Delta| \ll E_F$ (we shall see that $\Delta \leq 2w_D$). Thus for $\xi_k \sim \Delta$

$$\frac{D_s(E_k)}{D_n(E_F)} = \frac{d\xi_x}{dE_k} = \frac{d}{dE_k}\sqrt{E_k^2 - \Delta^2} = \frac{E_k}{\sqrt{E_k^2 - \Delta^2}} \qquad E_k > \Delta$$



$$\Delta = \frac{V_0}{L^3} \sum_k \sin\theta_k \cos\theta_k \ = \frac{V_0}{L^3} \sum_k u_k v_k = \frac{V_0}{L^3} \sum_k \frac{\Delta}{2E_k}$$

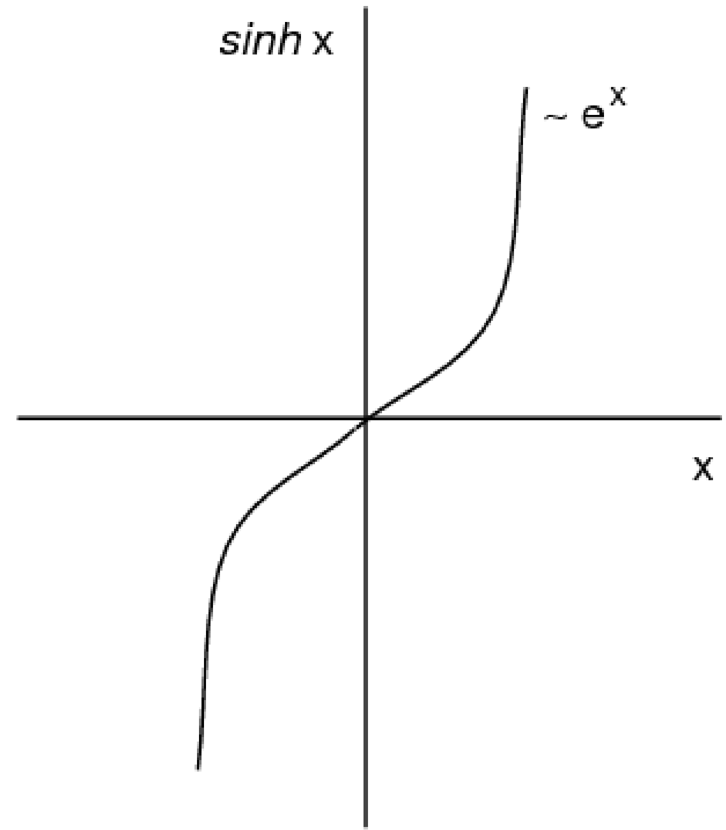
$$\Delta \ = \frac{1}{2} \frac{V_0}{L^3} \sum_k \frac{\Delta}{\sqrt{\xi_k^2 + \Delta^2}}$$

Convert this to sum over energy states (at T = 0 all states with $\xi < 0$ are occupied since $\xi_k = \hbar^2 k^2/2m - E_F$.

$$\Delta = \frac{V_0}{2}\Delta \int_{-\hbar\omega_D}^{\hbar\omega_D} \frac{Z(E_F+\xi)d\xi}{\sqrt{\xi^2+\Delta^2}}$$

$$\frac{1}{V_0 Z(E_F)} = \int_0^{\hbar\omega_D} \frac{d\xi}{\sqrt{\xi^2+\Delta^2}}$$

$$\frac{1}{V_0 Z(E_F)} = \sinh^{-1}\left(\frac{\hbar\omega_D}{\Delta}\right)$$



For small Δ,

$$\frac{\hbar\omega_D}{\Delta} \sim e^{\frac{1}{V_0 Z(E_F)}}$$

$$\Delta \simeq \hbar\omega_D e^{-\frac{1}{V_0 Z(E_F)}}$$

Consequences of BCS and Experiment

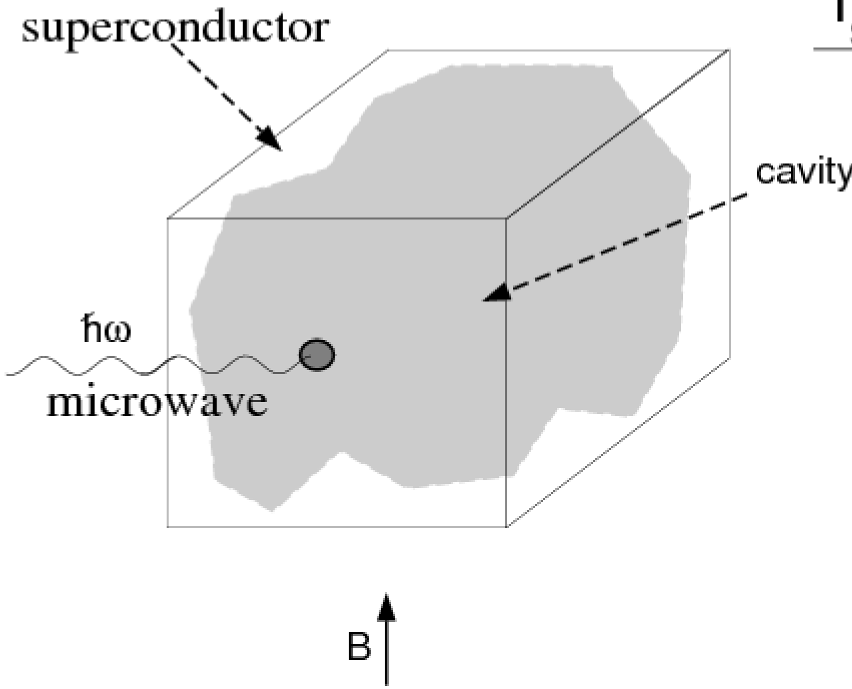
Specific Heat

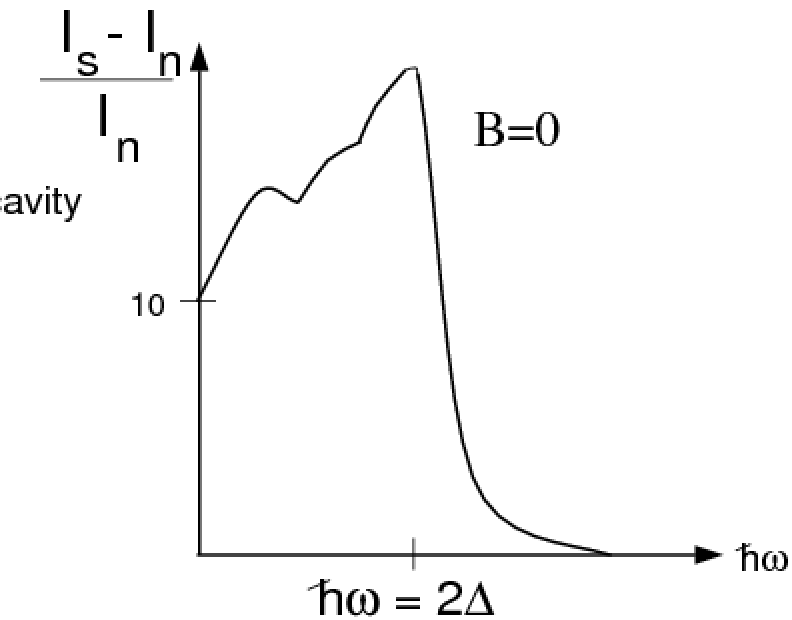
The simplest excitation which can be induced in a superconductor has energy 2Δ. Thus

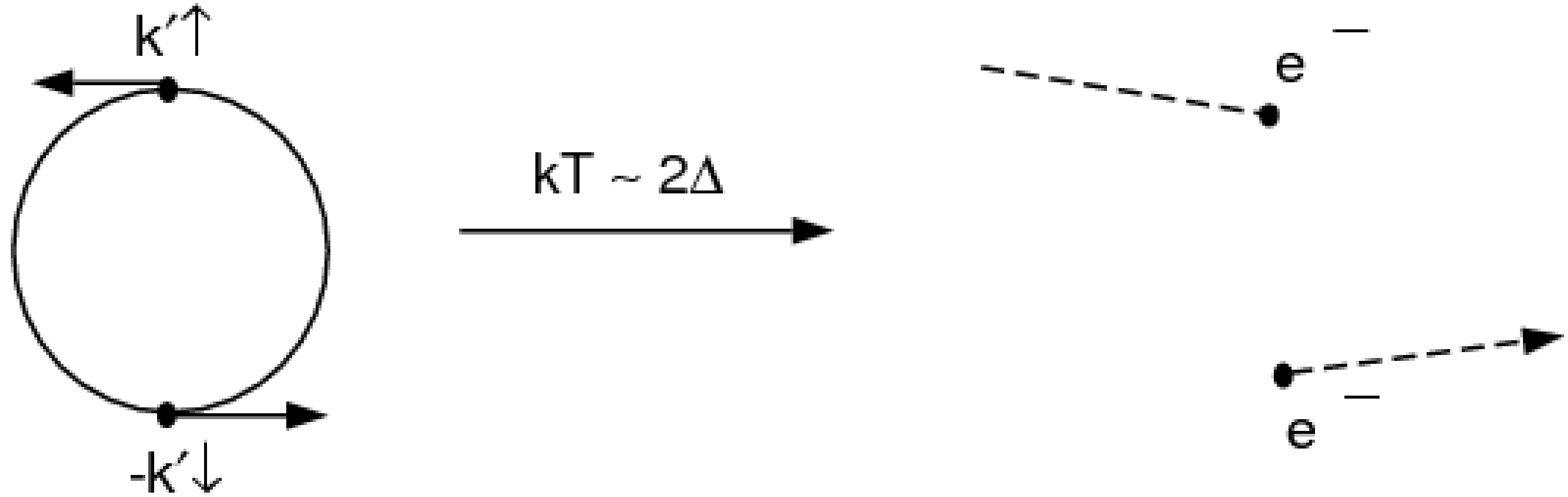
$$\Delta E \sim 2\Delta e^{-\beta 2\Delta} \qquad T \ll T_c$$

$$C \sim \frac{\partial \Delta E}{\partial \beta}\frac{\partial \beta}{\partial T} \sim \frac{\Delta^2}{T^2} e^{-\beta 2\Delta}$$

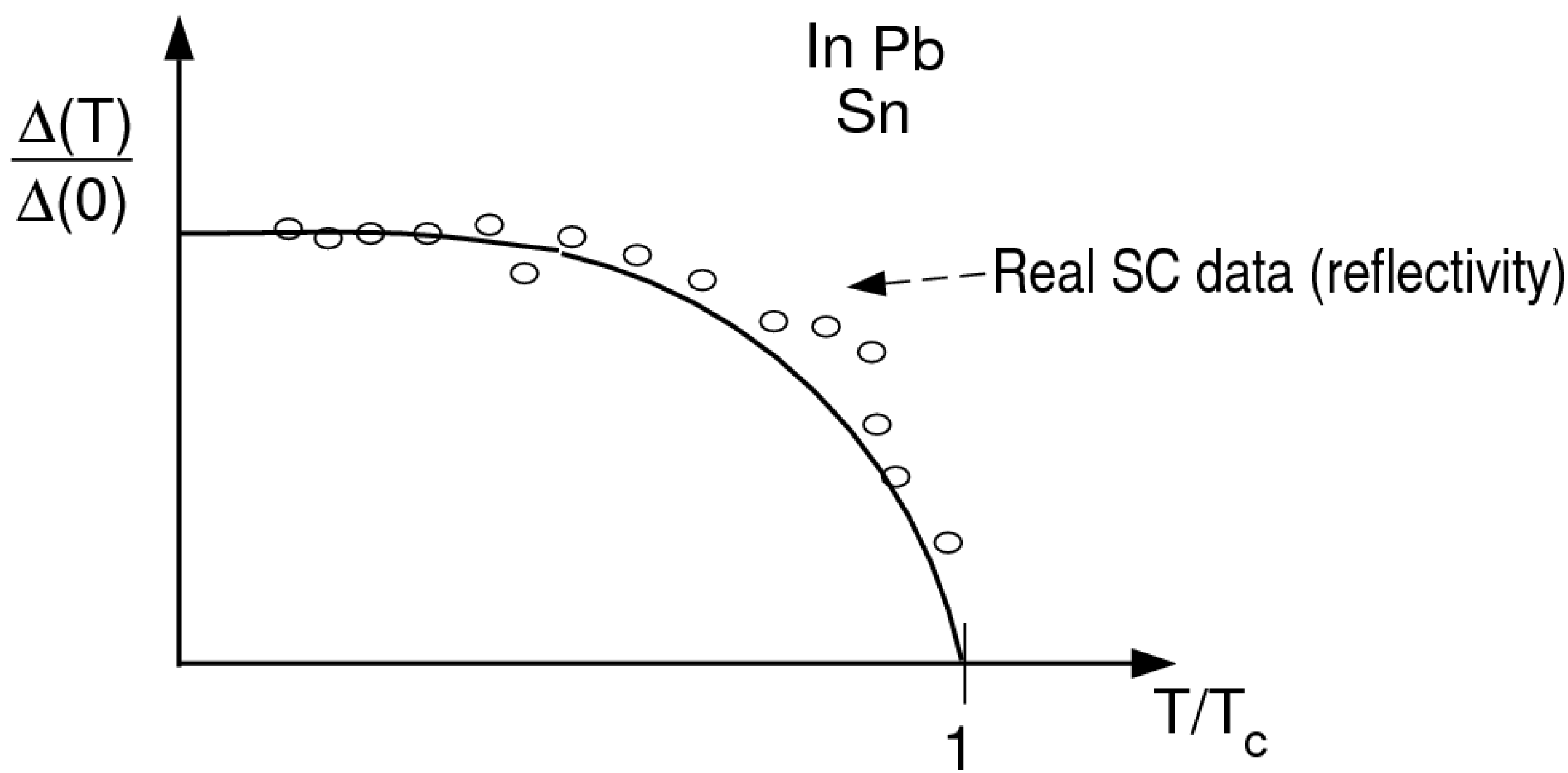
Microwave Absorption and Reflection



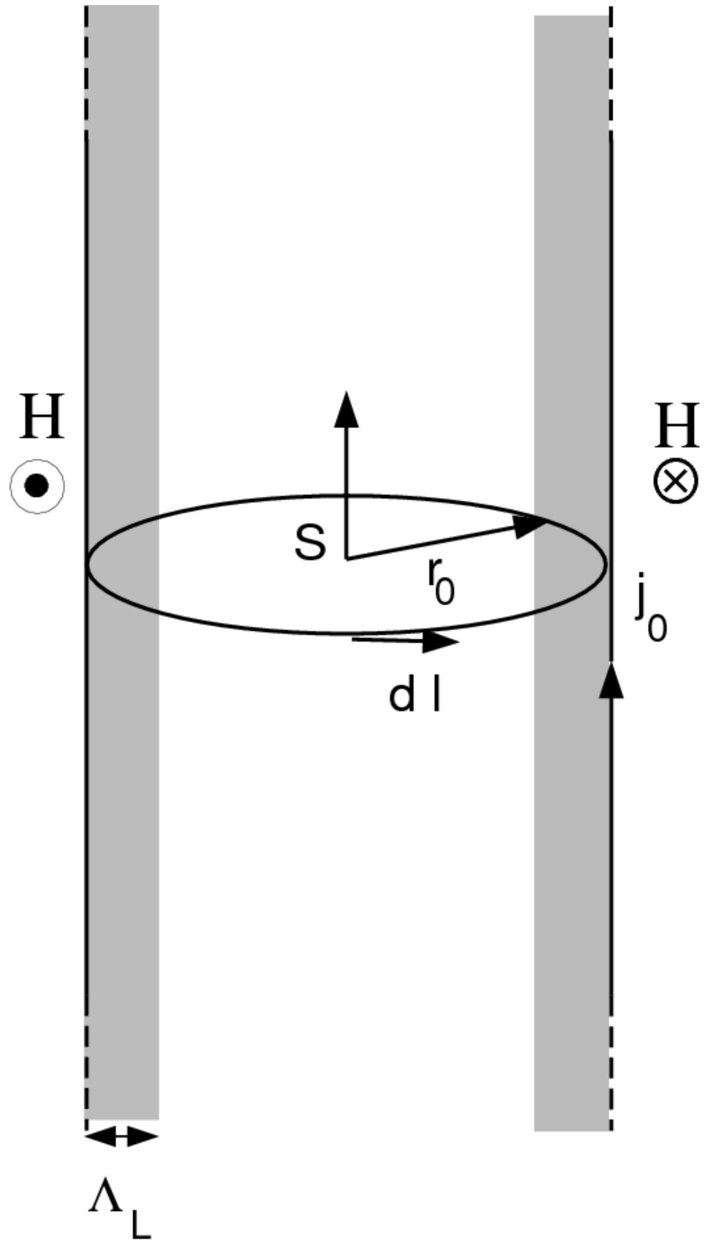




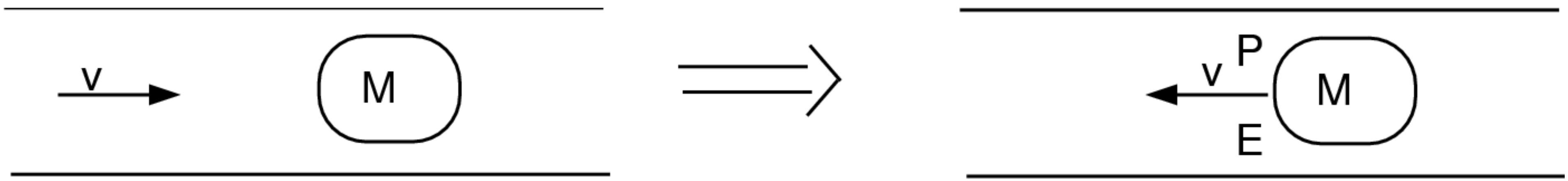
k′↑
-k′↓
kT ~ 2Δ
e⁻
e⁻

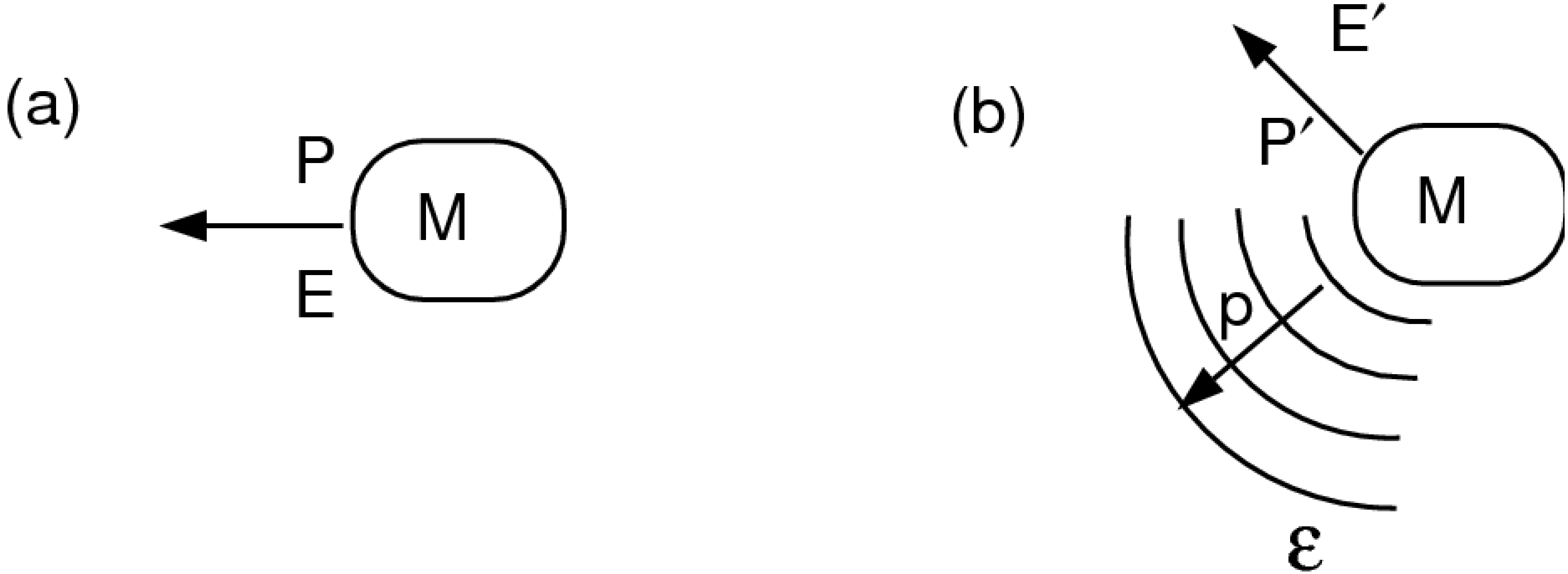


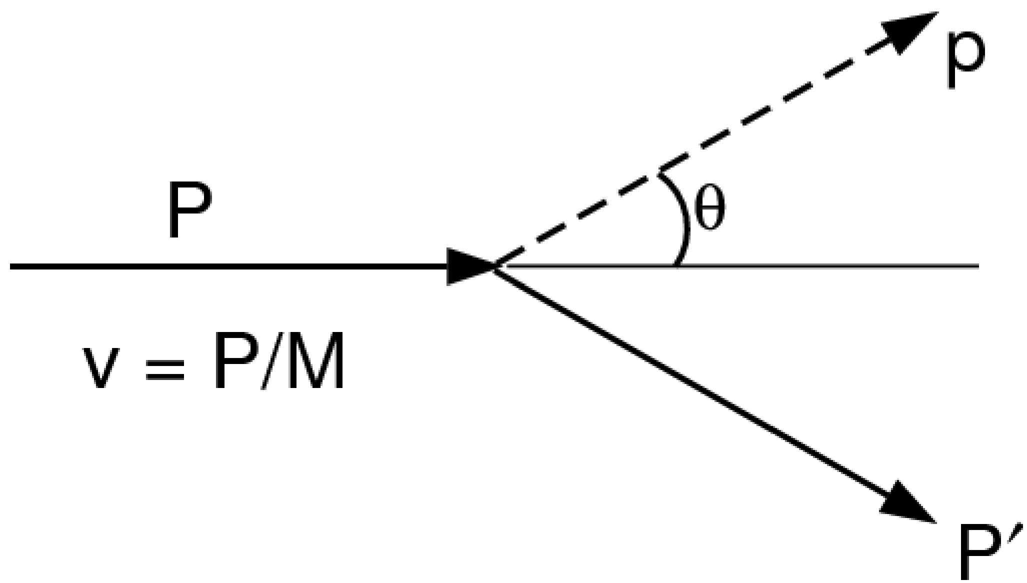
Δ(T)/Δ(0)
In Pb
Sn
Real SC data (reflectivity)
T/T_c
1

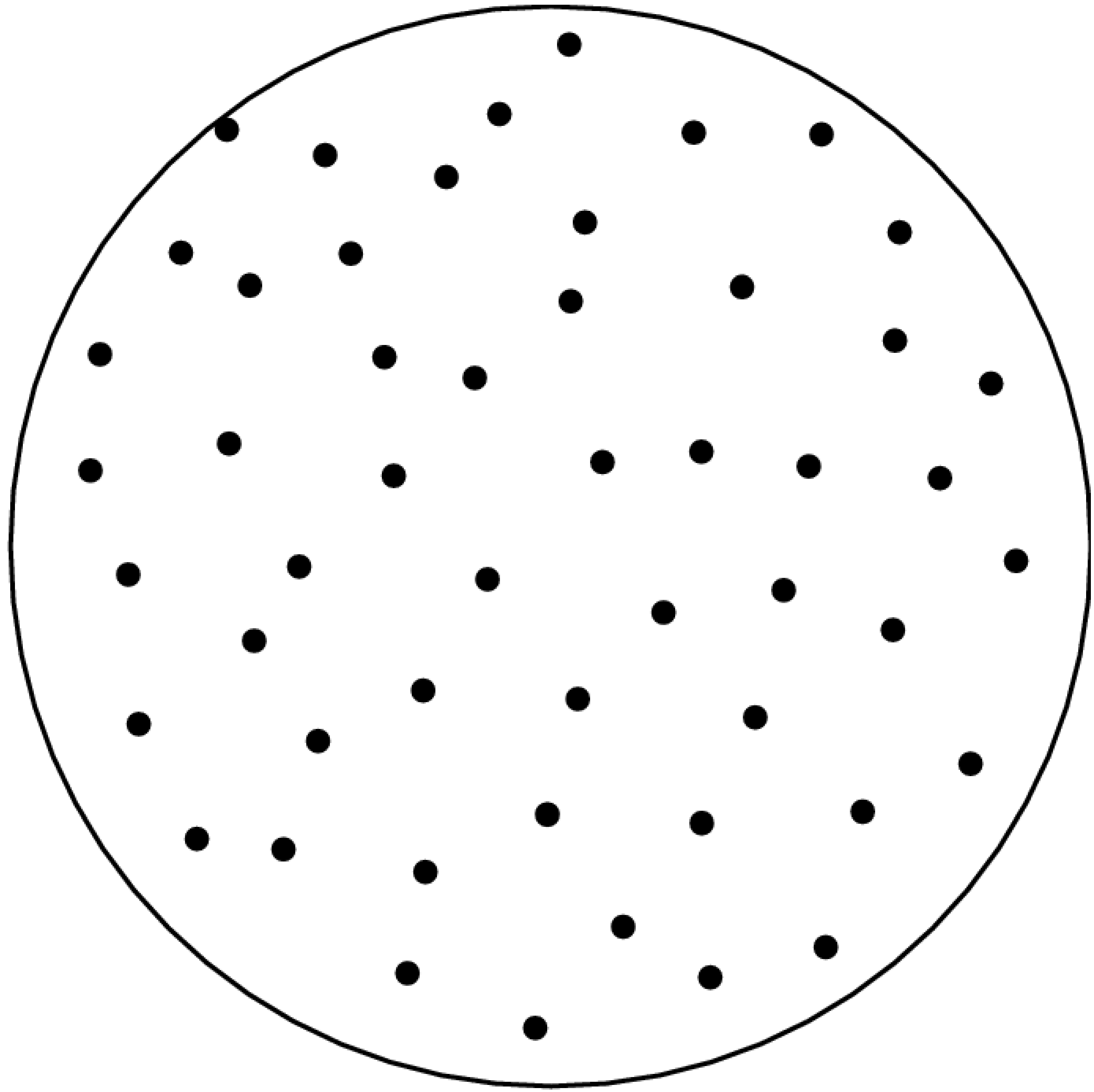


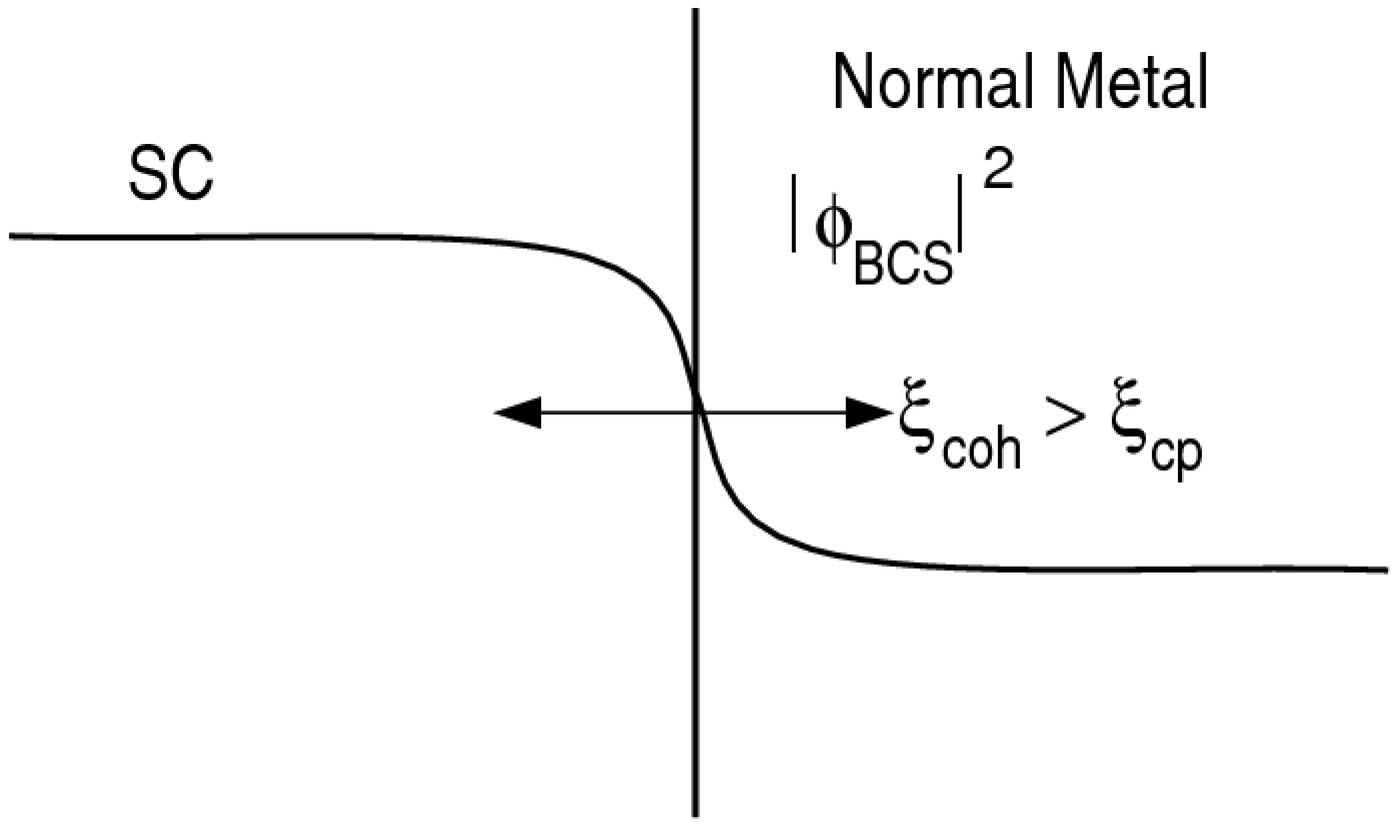
$$j = j_0 \, e^{(r - r_0)/\Lambda_L}$$

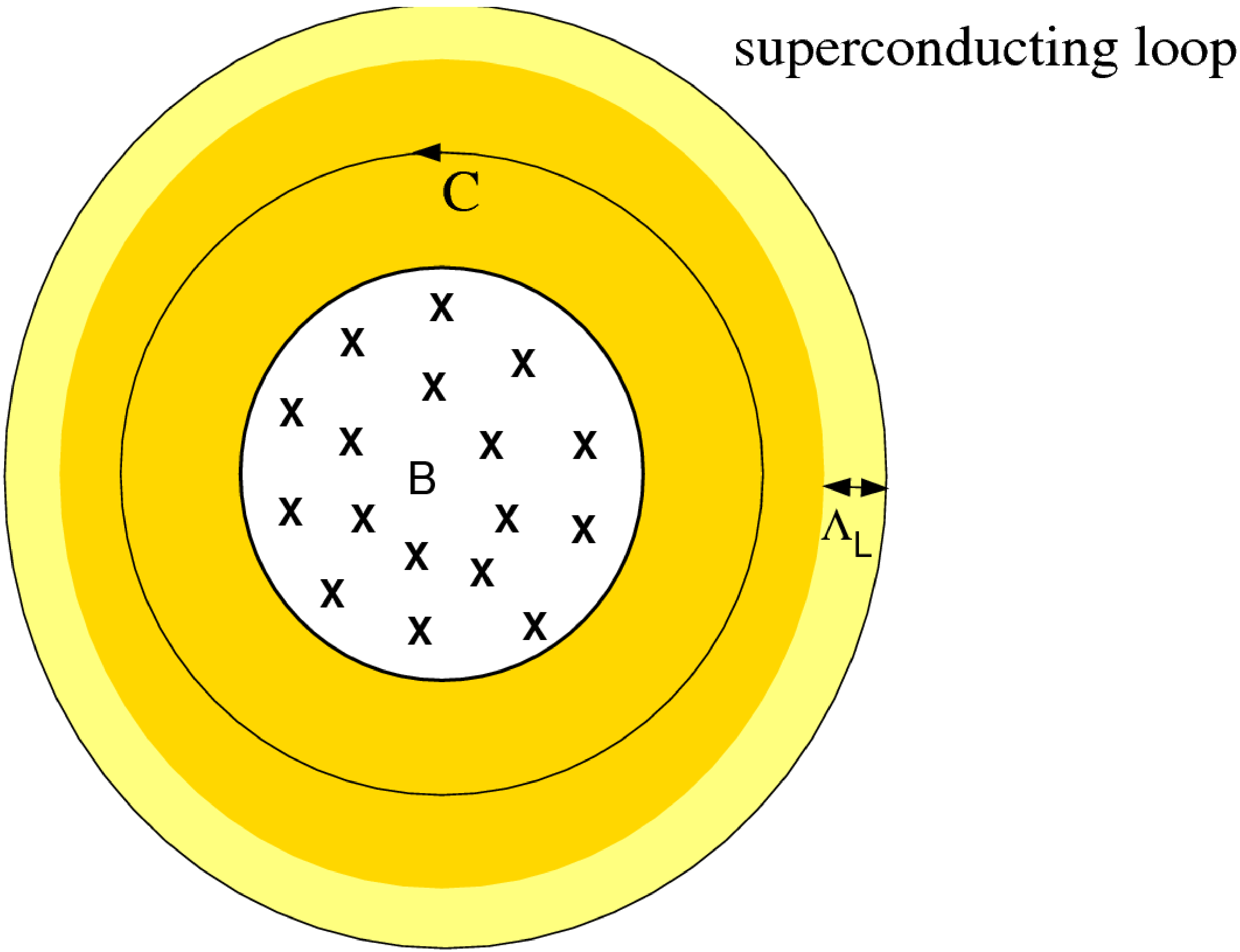

v
M
v
P
M
E

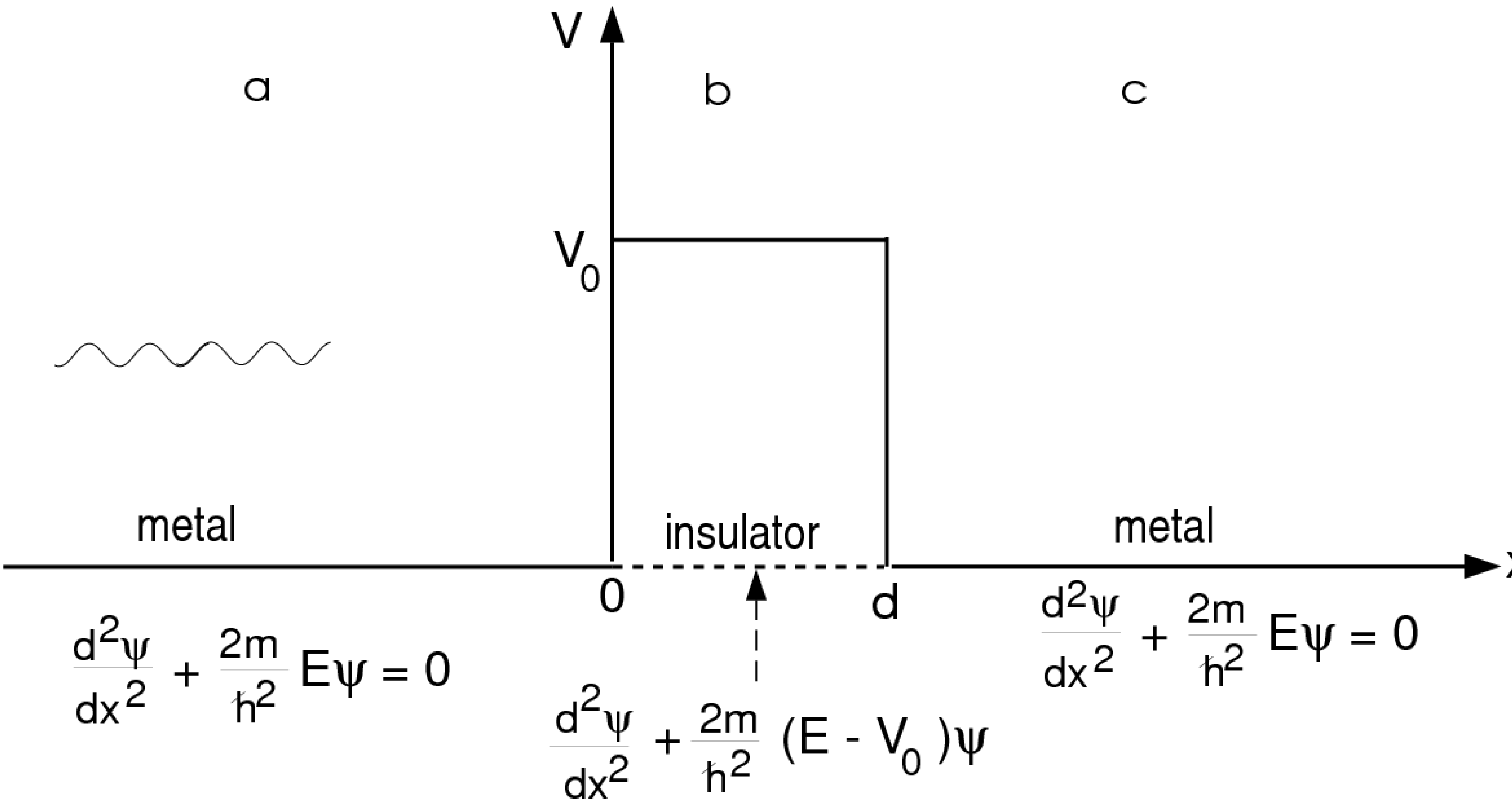

(a)
P
M
E
(b)
E′
P′
M
p
ε


p
P
θ
v = P/M
P′

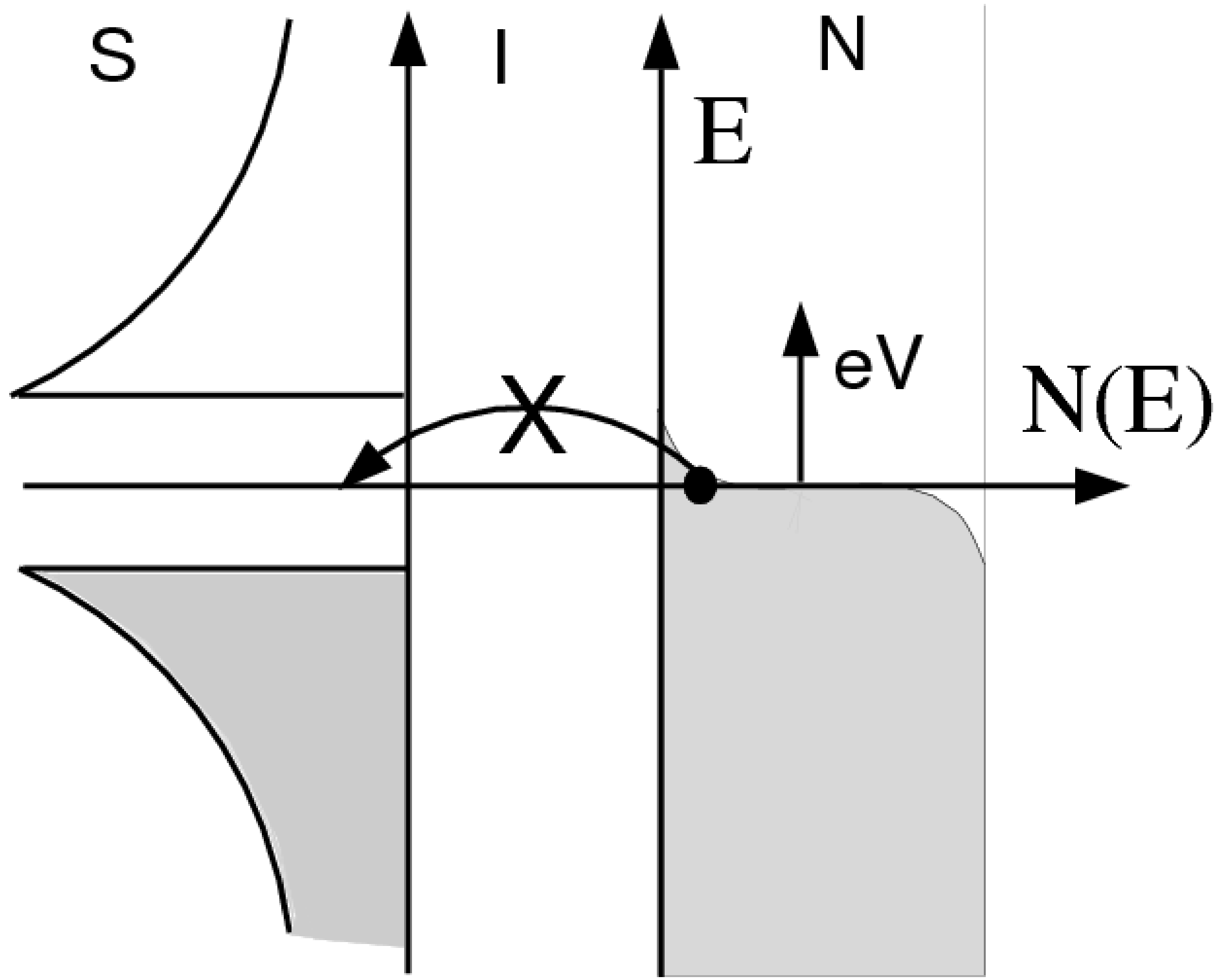


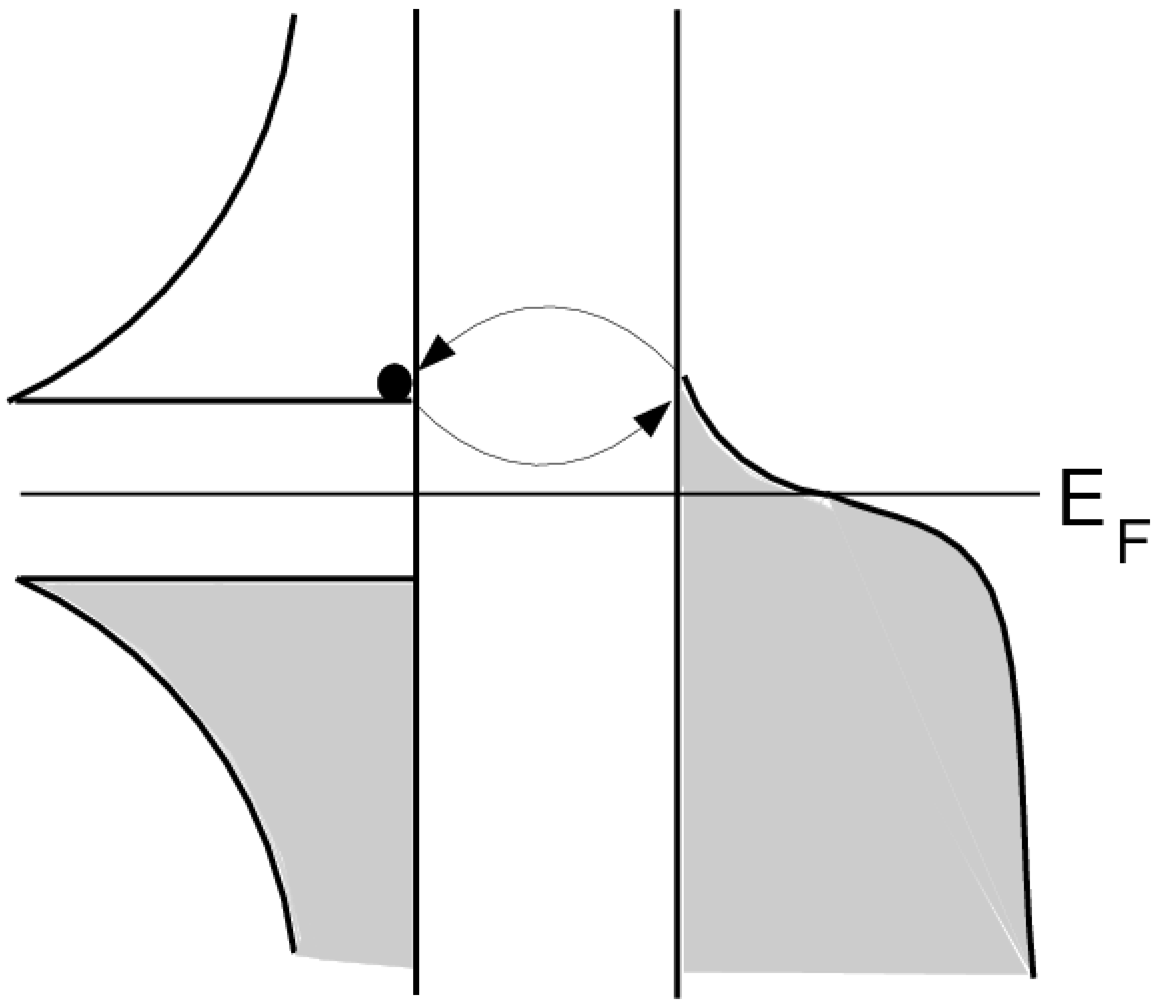

Normal Metal
SC
$|\phi_{BCS}|^2$
$\xi_{coh} > \xi_{cp}$


superconducting loop
C
B
Λ_L

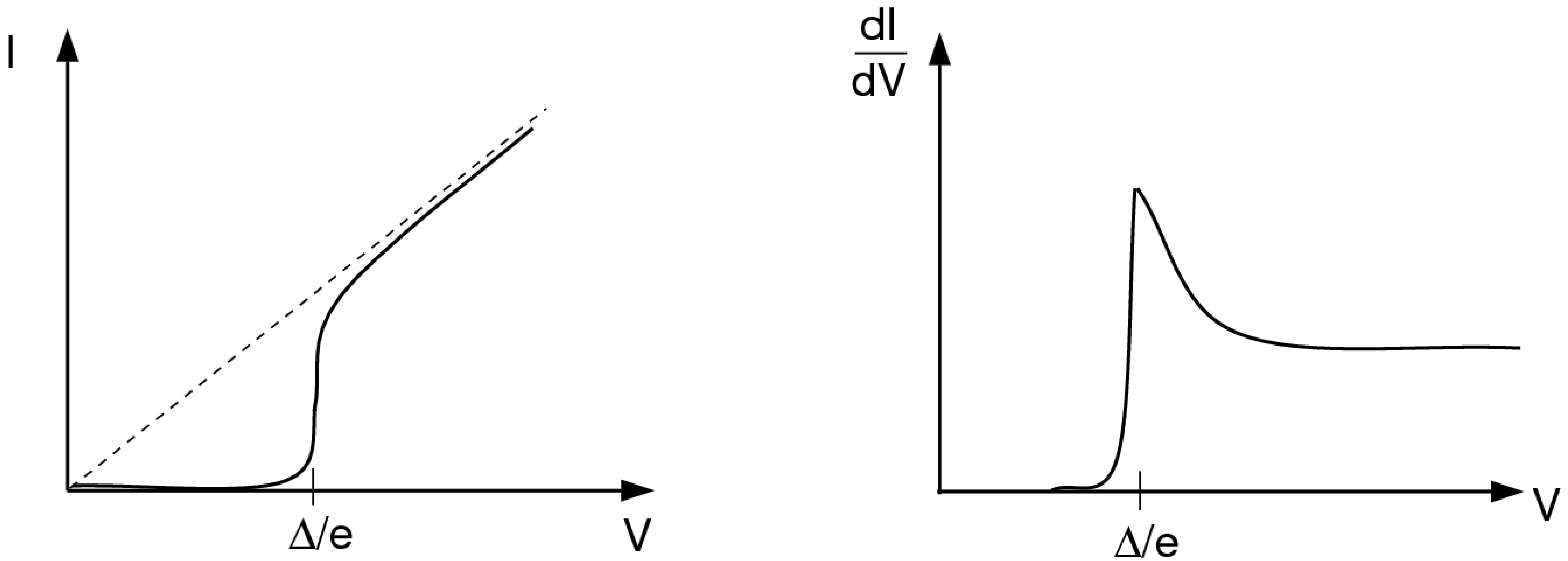


V
a
b
c
V_0
metal
insulator
metal
x
0
d
$\frac{d^2\psi}{dx^2} + \frac{2m}{\hbar^2} E\psi = 0$
$\frac{d^2\psi}{dx^2} + \frac{2m}{\hbar^2} (E - V_0)\psi$
$\frac{d^2\psi}{dx^2} + \frac{2m}{\hbar^2} E\psi = 0$


S
I
N
E
eV
N(E)
X



E_F


I
V
Δ/e
dI/dV
V
Δ/e