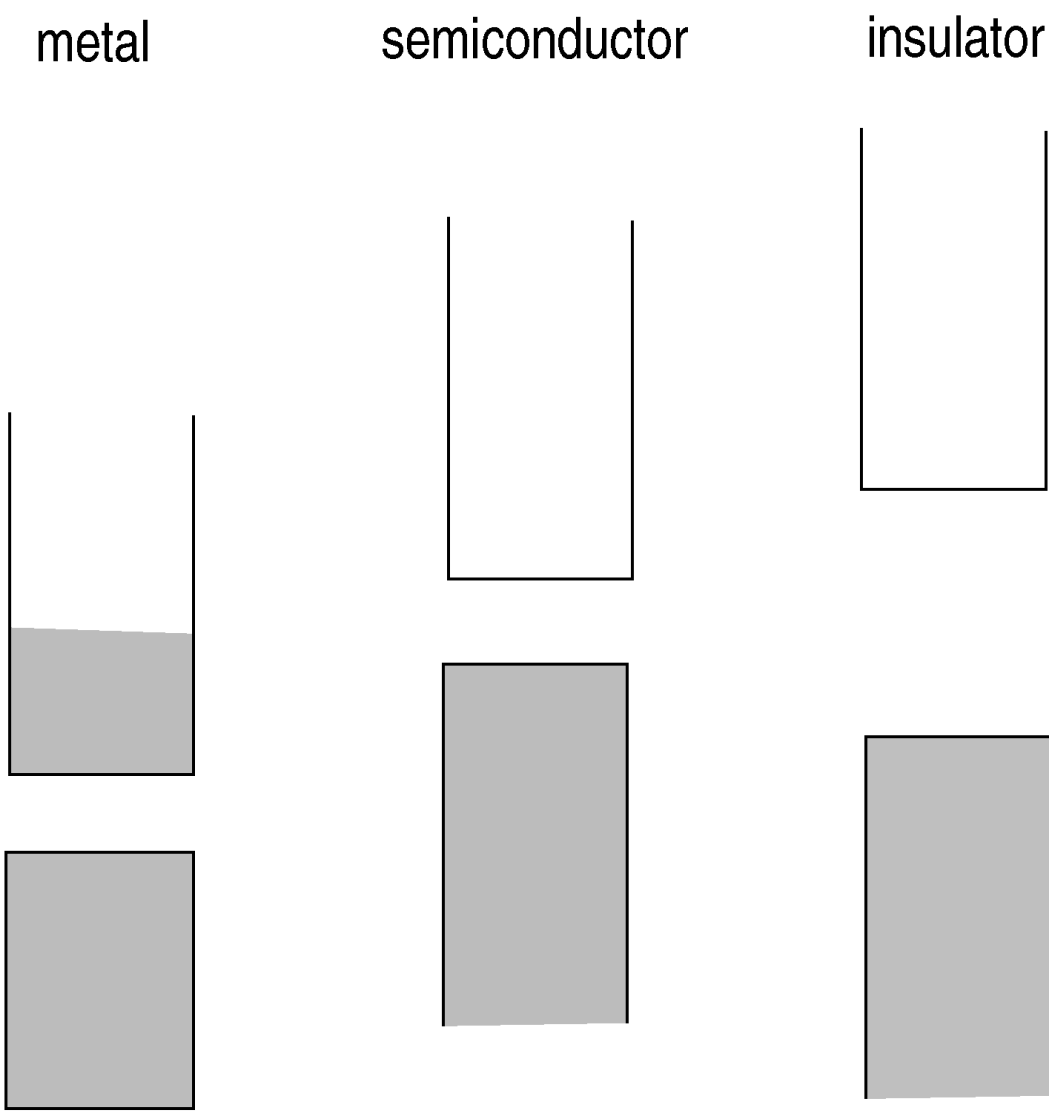
Semiconductors

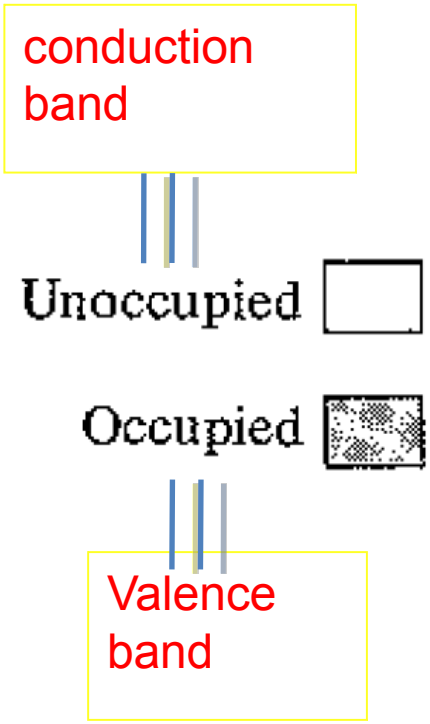
John Bardeen
(1908-1991)

Walter Schottky
(1886-1976)

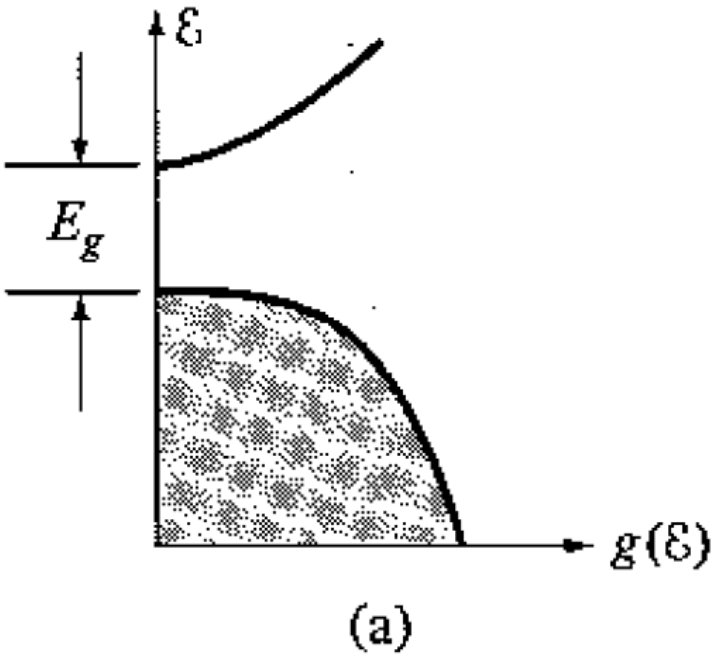
Band Structure

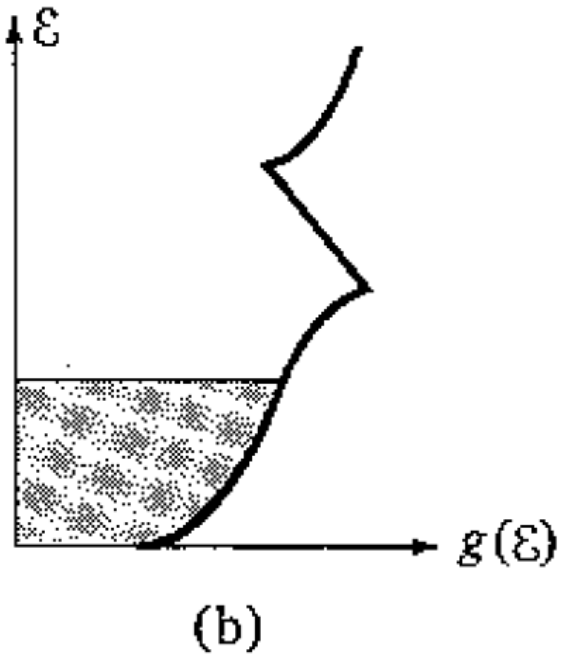


Where are the carriers coming from in semiconductors ?



Non-metallic **Metallic**





T = 0 K no conduction

T ≠ 0 K may/may not

What is a Semiconductor?

Most condensed matter physicists make the distinction on the basis of the conductivity and its temperature dependence. In the Drude model (parabolic band)

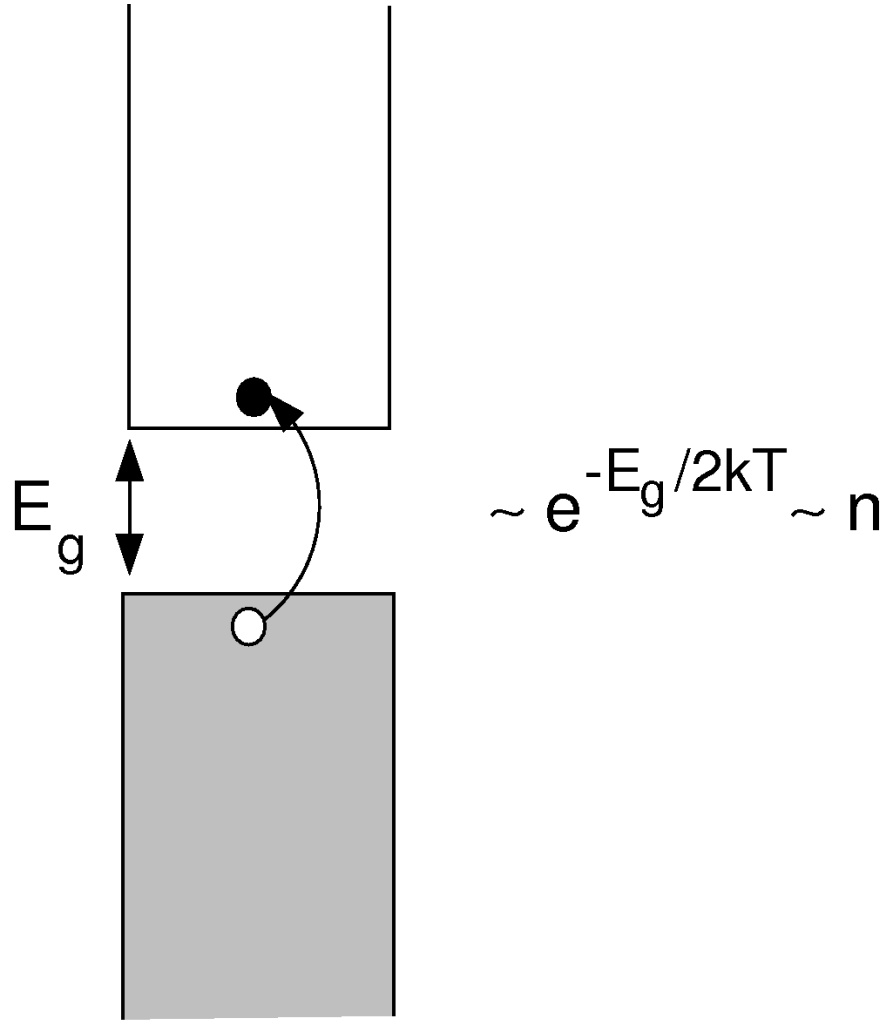
$$\sigma = \frac{ne^2\tau}{m^*}, \qquad \mu = \frac{e\tau}{m^*}, \qquad \sigma = ne\tau$$

$$\Rightarrow \text{metals} \qquad \sigma \sim \tau \sim \frac{1}{T}, \qquad \sigma \downarrow \text{ as } T \uparrow$$

In semiconductors, the population of free carriers *n* is temperature dependent. The exponential always will dominate the power law dependence of T .

$$\sigma \sim \tau n \sim \frac{1}{T} e^{-E_g/kT}$$

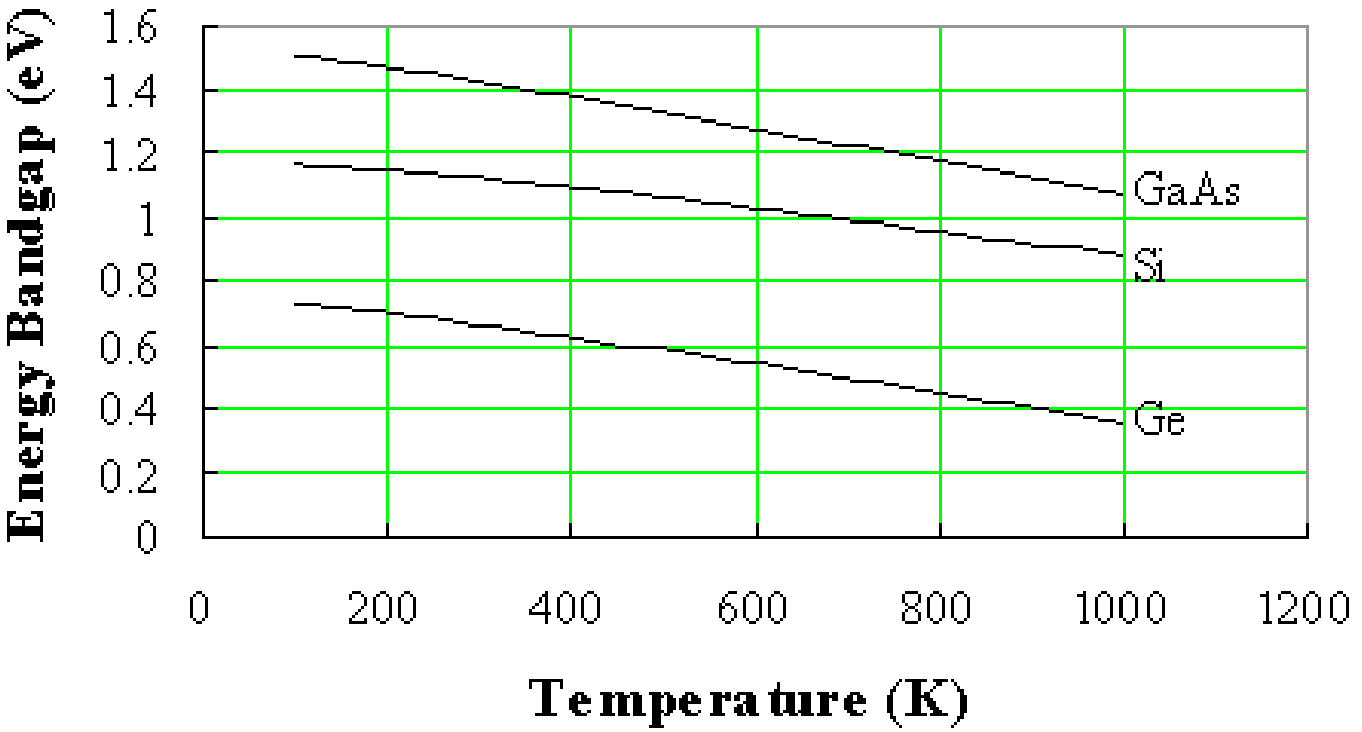
$$\sigma \uparrow \quad \text{as} \quad T \uparrow$$



The same is true for insulators, of course, except here n is so small that for all realistic purposes σ ~ 0.

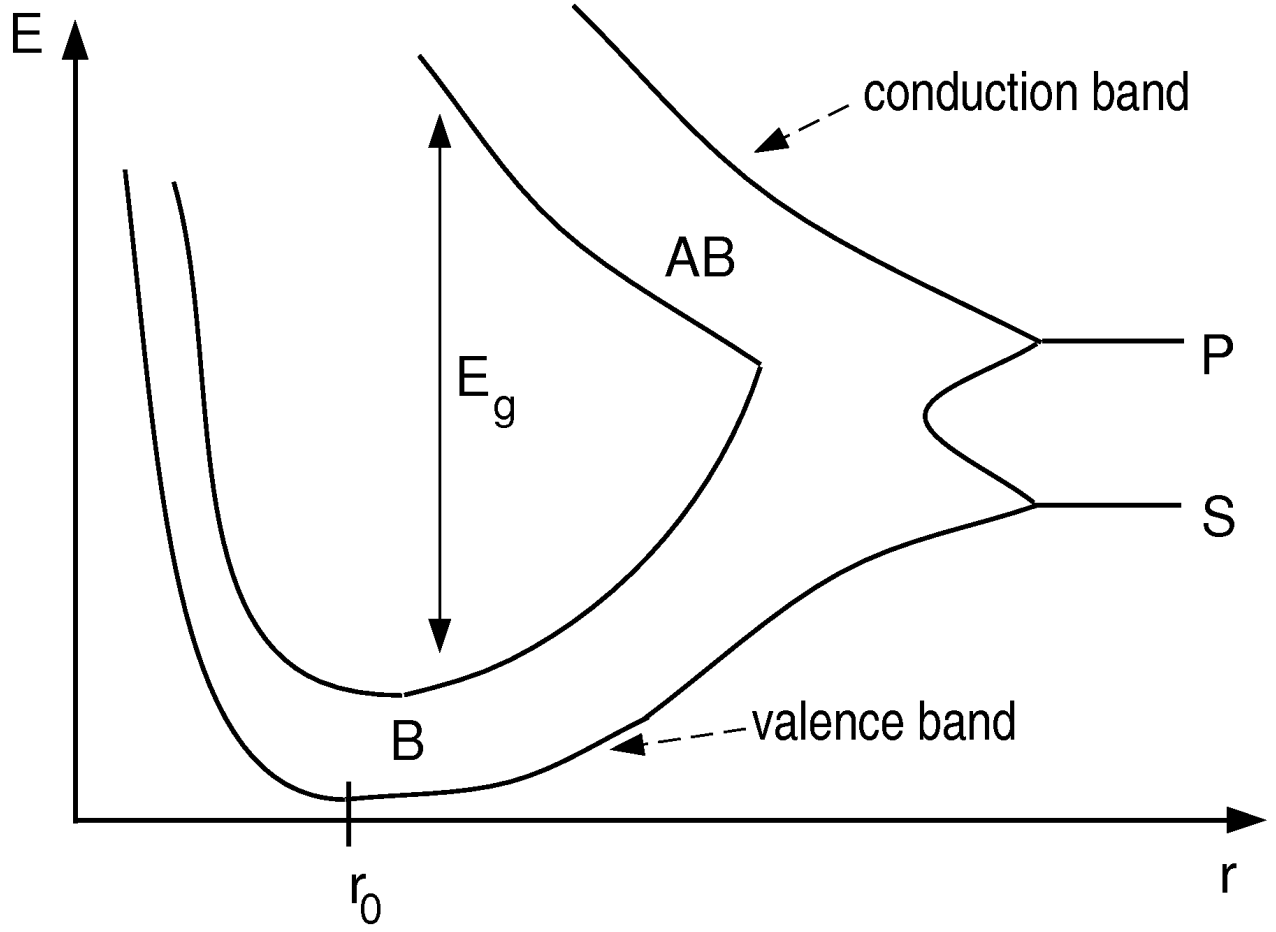
Temperature Dependence of Band Gaps

$$E_g(T) = E_g(0) - \frac{\alpha T^2}{T + \beta}$$



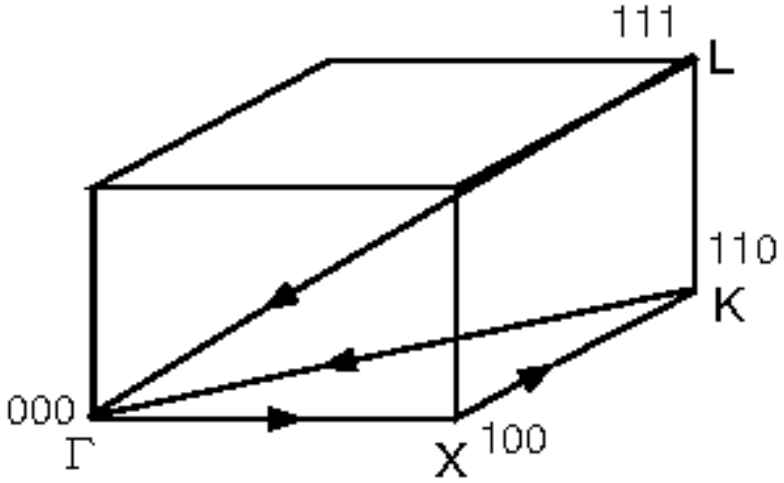
Increase of the inter-atomic spacing leads to the decrease of potential

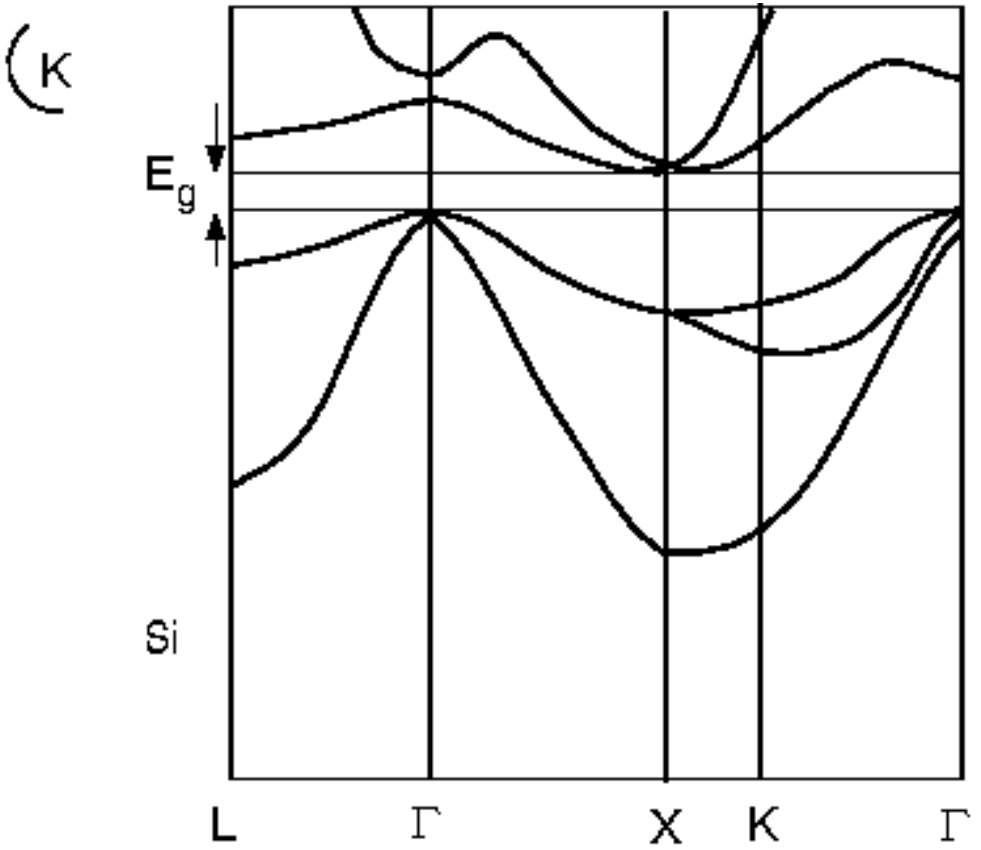
Sketch of the sp^3 bands in Si vs. Si-Si separation



$E_g \downarrow \quad T \downarrow$

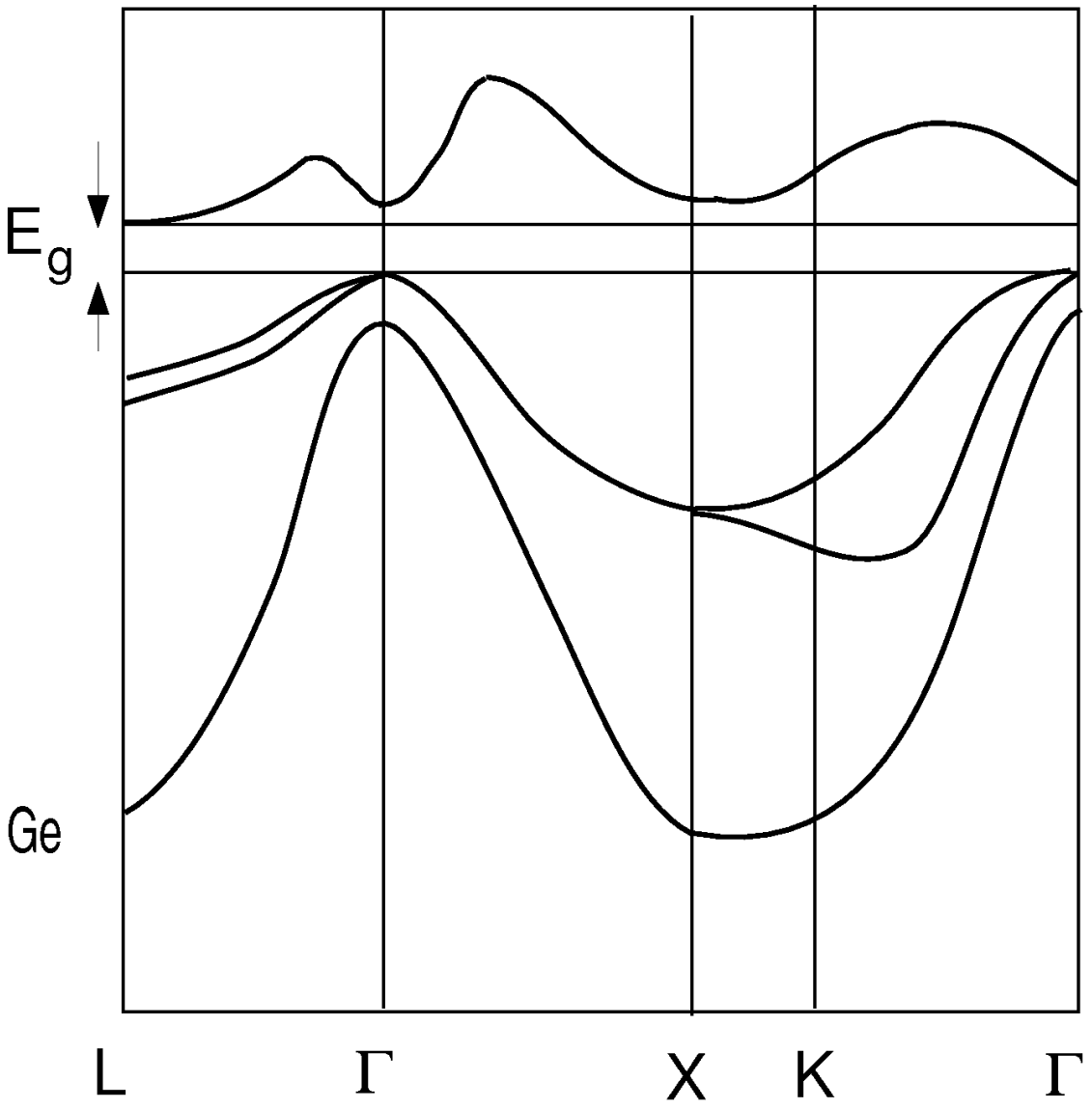
$sp^3 \Rightarrow$ 4 electrons per band

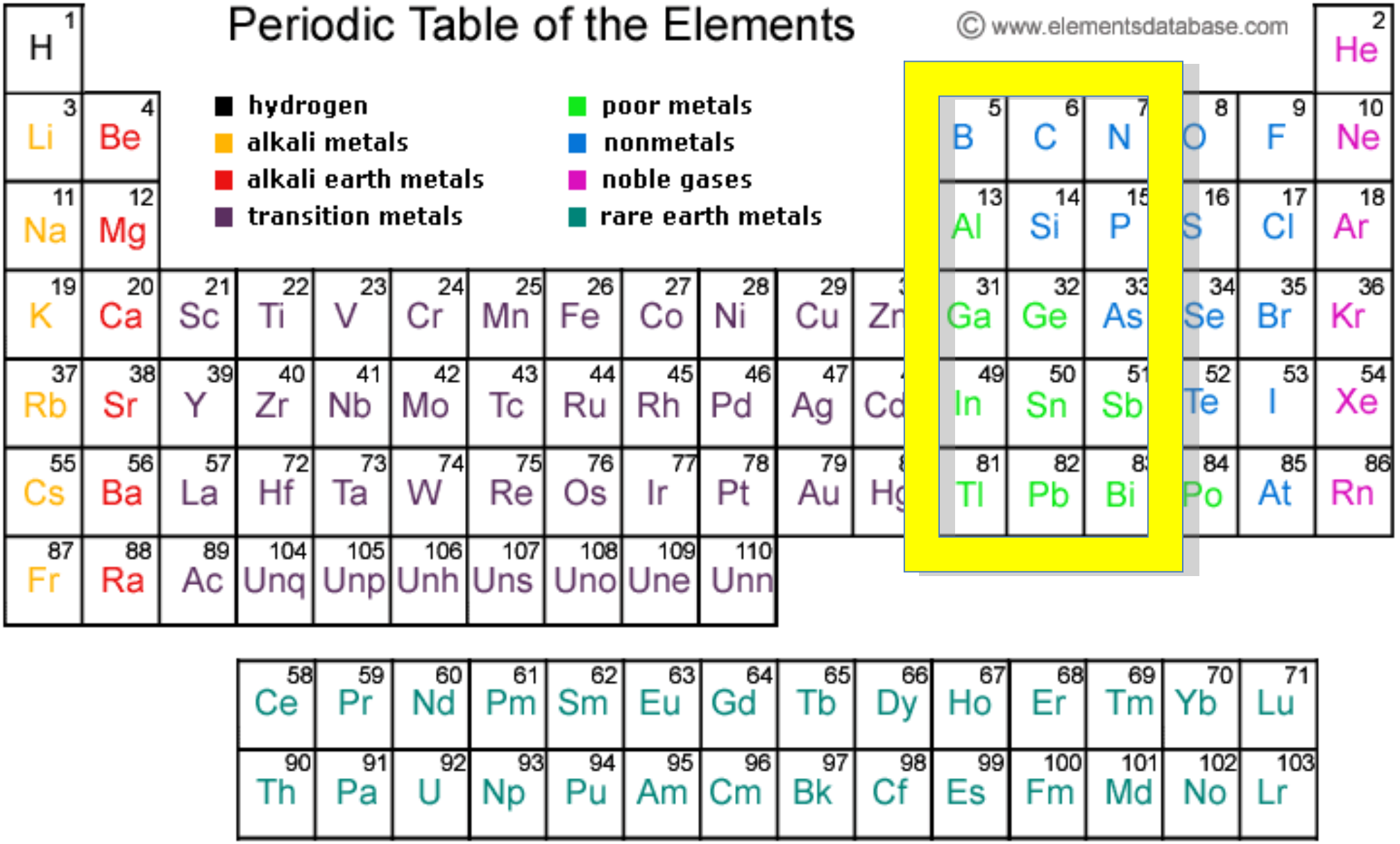

111
L
110
K
000
Γ
X
100


K
E_g
Si
L
Γ
X
K
Γ

Indirect, roughly Γ → L,
minimum gap energy

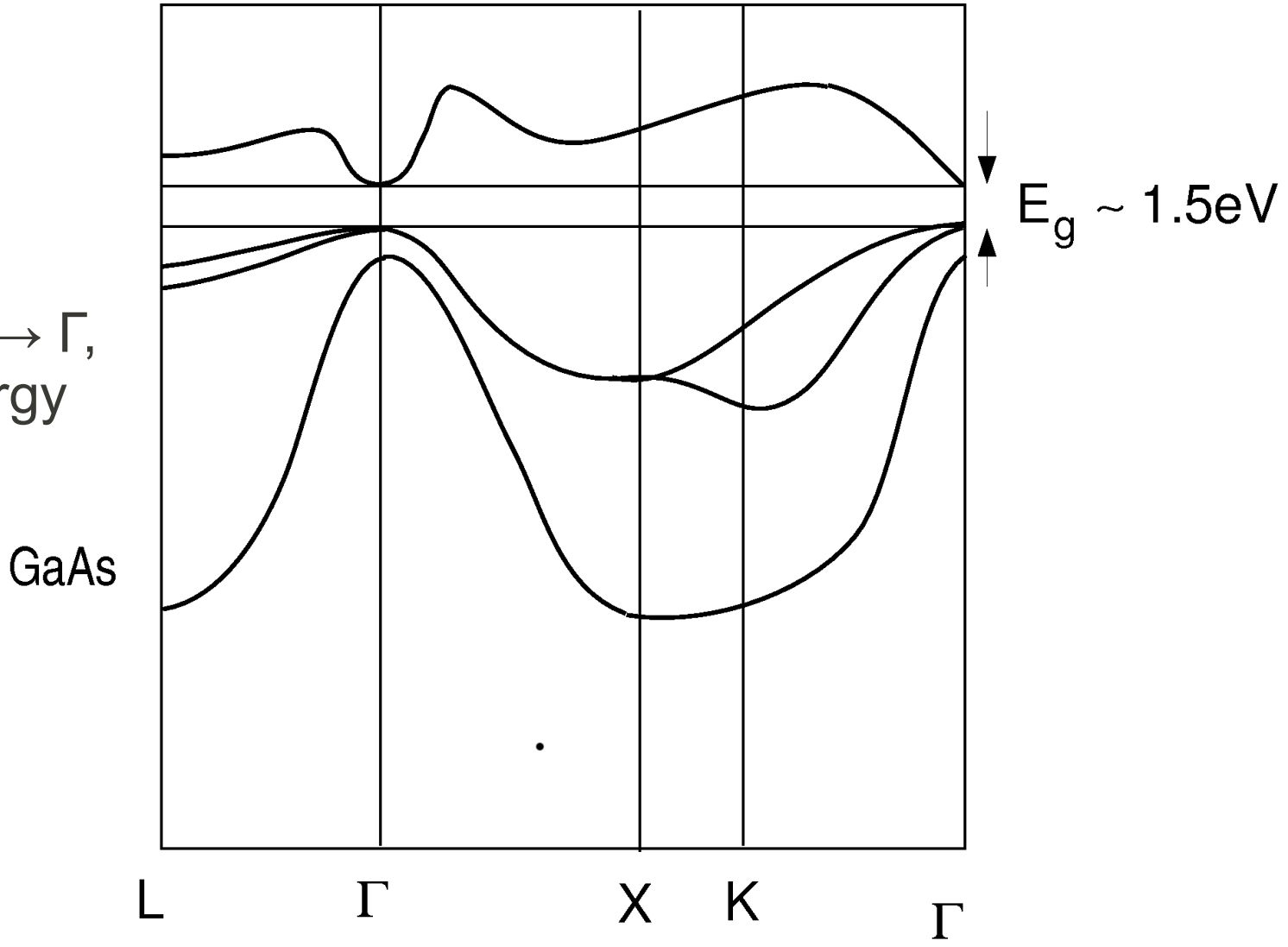
$$\frac{1}{m^*_{ij}} = \frac{1}{\hbar^2}\frac{\partial^2 E(\mathbf{k})}{\partial k_i \partial k_j}$$





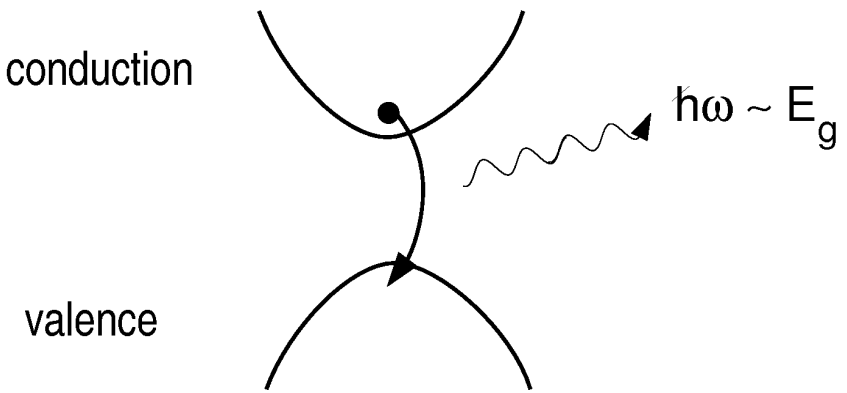
Most semiconductors have co-valiant bonding

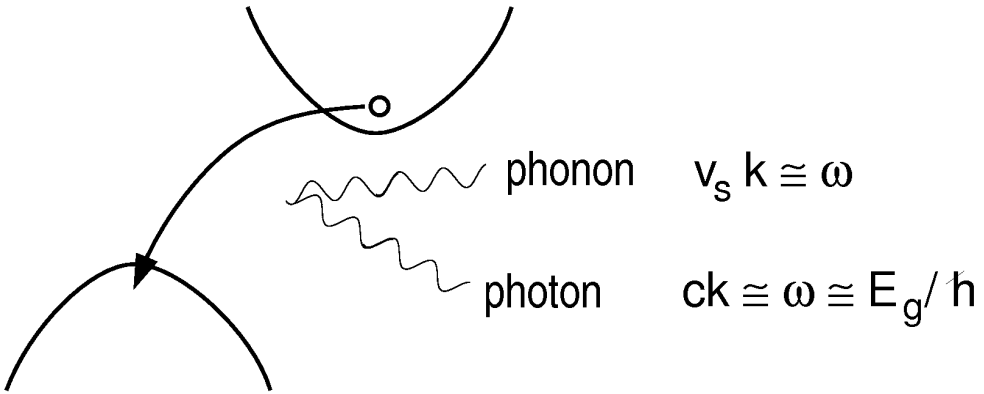
Note the direct, $\Gamma \rightarrow \Gamma$, minimum gap energy



material	τ_{exciton}
GaAs	$1ns(10^{-9}s)$
Si	$19\mu s(10^{-5}s)$
Ge	$1ms(10^{-3}s)$

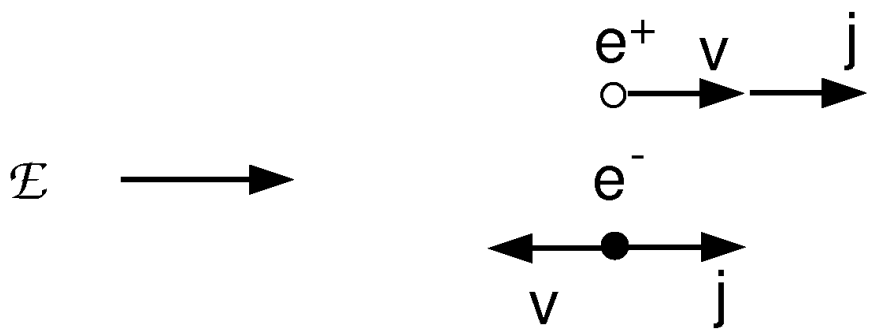
Direct vs. In-direct gap





Charge Carrier Density in Intrinsic Semiconductors

Both electrons and holes contribute to the conductivity



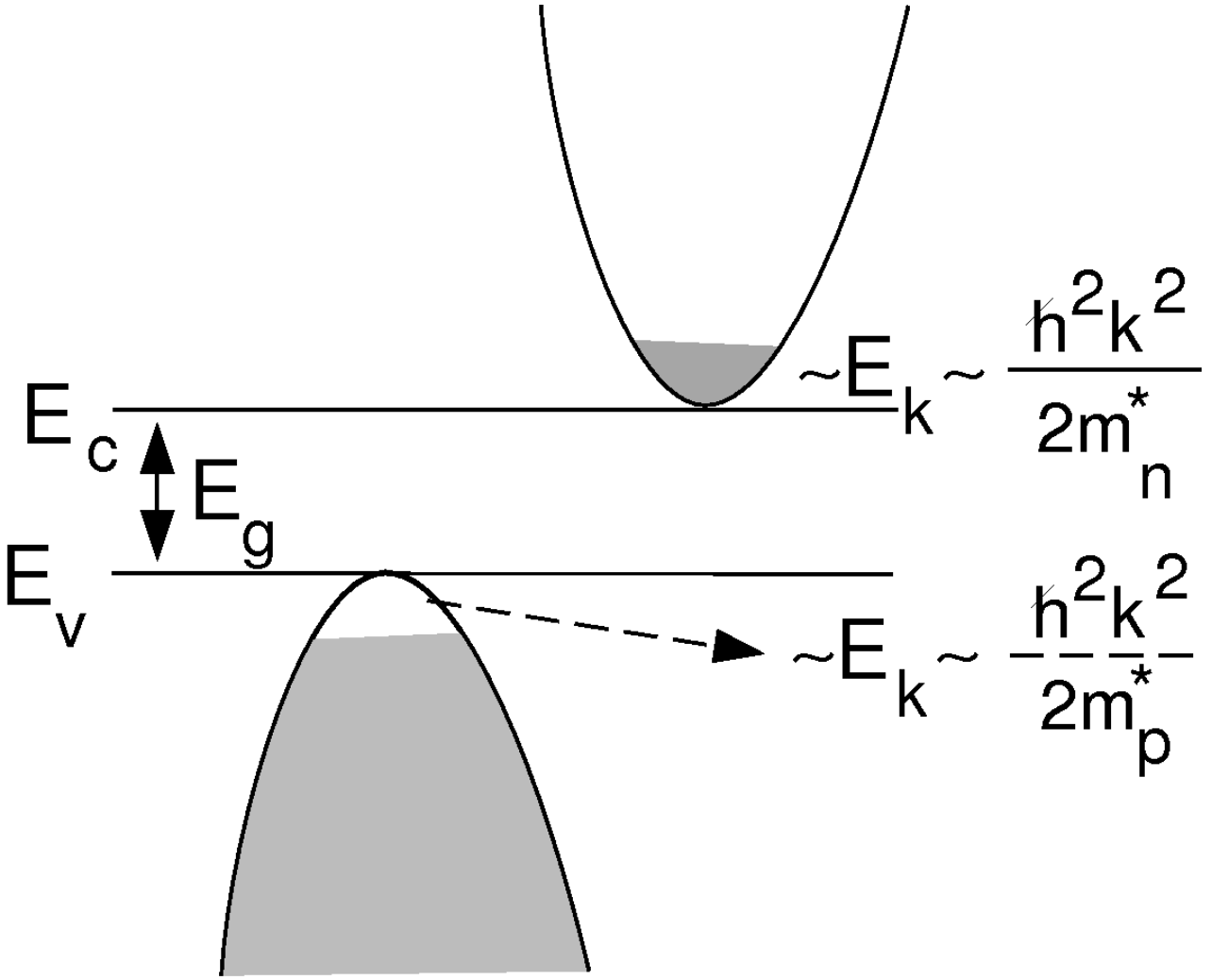
$$n = \frac{\#\ \text{electrons}}{\text{volume}}$$

$$p = \frac{\#\ \text{holes}}{\text{volume}}$$

$$\sigma = e(n\mu_n + p\mu_r)$$

Mobilities are assumed to be constant: all of the conducting carriers are full near the top or bottom of bands, where $E_k \sim \hbar^2k^2/2m^*$ and the effective mass approximation is valid $\mu \sim e\tau/m^*$

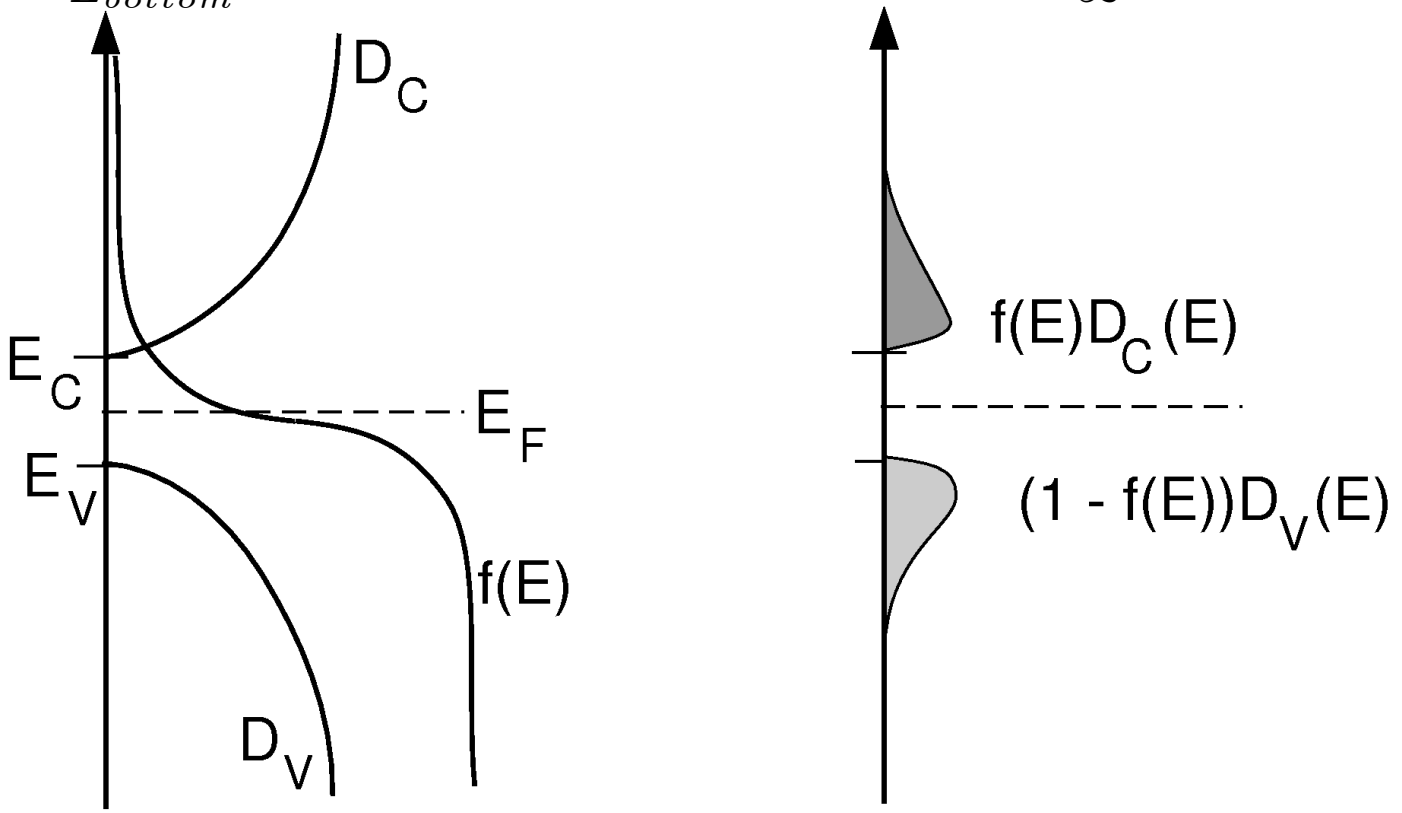
$$\mu \sim \frac{e\tau}{m^*}$$



The carrier concentrations are highly T -dependent since all of the carriers in an intrinsic (un-doped) semiconductor are thermally induced (i.e. n = p = 0 at T = 0)

$$n = \int_{E_c}^{E_{top}} D_C(E) f(E,T) dE \rightarrow \int_{E_c}^{\infty} D_C(E) f(E,T) dE$$

$$p = \int_{E_{bottom}}^{E_v} D_V(E) \{1 - f(E,T)\} dE \rightarrow \int_{-\infty}^{E_v} D_V(E) \{1 - f(E,T)\} dE$$



need forms for D_c and D_V

For the parabolic approximation $E_k \simeq \frac{\hbar^2 k^2}{2m^*}$ we have

$$D(E) = \frac{(2m^*)^{\frac{3}{2}}}{2\pi^2\hbar^3}\sqrt{E}$$

$$\Rightarrow \quad D_C(E) = \frac{(2m_n^*)^{\frac{3}{2}}}{2\pi^2\hbar^3}\sqrt{E - E_C}$$

$$D_V(E) = \frac{(2m_p^*)^{\frac{3}{2}}}{2\pi^2\hbar^3}\sqrt{E_V - E}$$

Intrinsic (undoped) semiconductor n = p ⇒ EF must lie in the band gap.

However, if $m_p^* \neq m_n^*$ (ie. $D_C \neq D_V$) ⇒ the chemical potential, EF , must be adjusted up or down from the center of the gap so that n = p.

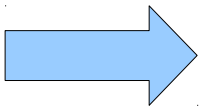
Furthermore, the carriers which are induced across the gap are relatively high in energy, compared to $k_B T$, since typically $E_g = E_C - E_V \gg k_B T$.

	$E_g(eV)$	$n_i(cm^{-3})(300^\circ K)$
Ge	0.67	2.4×10^{13}
Si	1.1	1.5×10^{10}
GaAs	1.43	5×10^{7}

$$\frac{1eV}{k_B} \sim 10000^\circ K \gg 300^\circ K \sim T$$

Thus, assuming that $E - E_F > \frac{E_g}{2} \gg k_B T$

$$\frac{1}{e^{(E-E_F)/k_B T} + 1} \simeq \frac{1}{e^{(E-E_F)/k_B T}} = e^{-(E-E_F)/k_B T}$$



Boltzmann statistics

A similar relationship holds for holes where $-(E - E_F) > \frac{E_g}{2} \gg k_B T$

$$1 - \frac{1}{e^{(E-E_F)/k_B T} + 1} \simeq 1 - \left\{1 - e^{-(E-E_F)/k_B T}\right\} = e^{-(E-E_F)/k_B T}$$

Since $(1 - f(E)) = f(-E)$ and $e^{(E-E_F)/k_B T}$ is small

$$\Rightarrow \quad n \simeq \frac{(2m_n^*)^{\frac{3}{2}}}{2\pi^2\hbar^3} e^{E_F/k_B T} \int_{E_C}^{\infty} \sqrt{E - E_C} e^{-E/k_B T} dE$$

$$= \frac{(2m_n^*)^{\frac{3}{2}}}{2\pi^2\hbar^3} (k_B T)^{\frac{3}{2}} e^{-\beta(E_C - E_F)} \int_0^{\infty} x^{\frac{1}{2}} e^{-x} dx$$

$$= 2\left(\frac{2\pi m_n^* k_B T}{h^2}\right)^{\frac{3}{2}} e^{-\beta(E_C - E_F)} = N_{eff}^C e^{-\beta(E_C - E_F)}$$

Similarly $p = 2\left(\frac{2\pi m_p^* k_B T}{h^2}\right)^{\frac{3}{2}} e^{-\beta(E_V - E_F)} = N_{eff}^V e^{-\beta(E_V - E_F)}$

In general, in the nondegenerate limit

$$np = 4\left(\frac{k_B T}{2\pi\hbar^2}\right)^3 \left(m_n^* m_p^*\right)^{\frac{3}{2}} e^{-\beta E_g}$$

➡ **Law of mass action** - holds for both doped and intrinsic semiconductor so long as we remain in the nondegenerate limit.

For an intrinsic semiconductor, where n = p:

$$n_i = p_i = 2\left(\frac{k_B T}{2\pi\hbar^2}\right)^{\frac{3}{2}} \left(m_n^* m_p^*\right)^{\frac{3}{4}} e^{-\beta E_g/2}$$

However, we already have relationships for n and p involving E_C and E_V

$$n = p = N_{eff}^C e^{-\beta(E_C - E_F)} = N_{eff}^V e^{\beta(E_V - E_F)}$$

$$e^{2\beta E_F} = \frac{N_{eff}^V}{N_{eff}^C} e^{\beta(E_V + E_C)}$$

$$E_F = \frac{1}{2}(E_V + E_C) + \frac{1}{2}k_B T \ln\left(\frac{N_{eff}^V}{N_{eff}^C}\right)$$

$$E_F = \frac{1}{2}(E_V + E_C) + \frac{3}{4}k_B T \ln\left(\frac{m_p^*}{m_n^*}\right)$$

Thus if $m_p^* \neq m_n^*$, the chemical potential EF in a semiconductor is temperature dependent.