Measuring the Fermi Surface

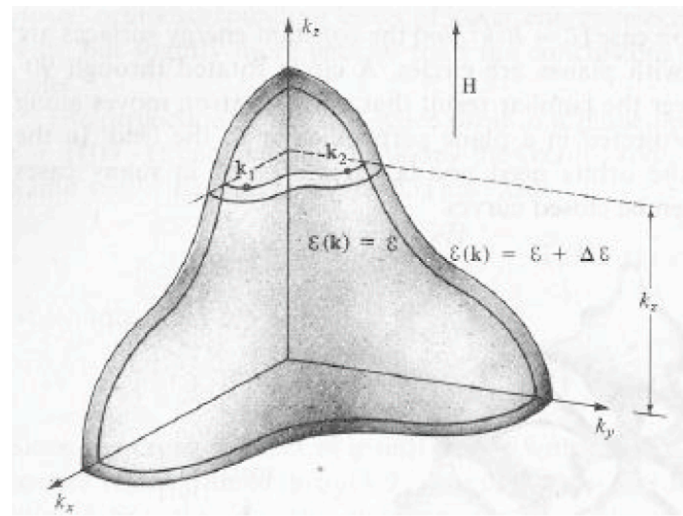
-- Physical Properties Depend on The Shape of the Fermi Surface

Recall: In the case of **E**(r,t)=0, **H**=H_0

$$T(\varepsilon, k_z) = \frac{\hbar^2 c}{eH} \frac{\Delta A(\varepsilon, k_z)}{\Delta \varepsilon}$$

$$m_c^*(\varepsilon, k_z) = \frac{\hbar^2}{2\pi} \underline{\frac{\Delta A(\varepsilon, k_z)}{\Delta \varepsilon}}$$

Determined by the shape of the Fermi surface

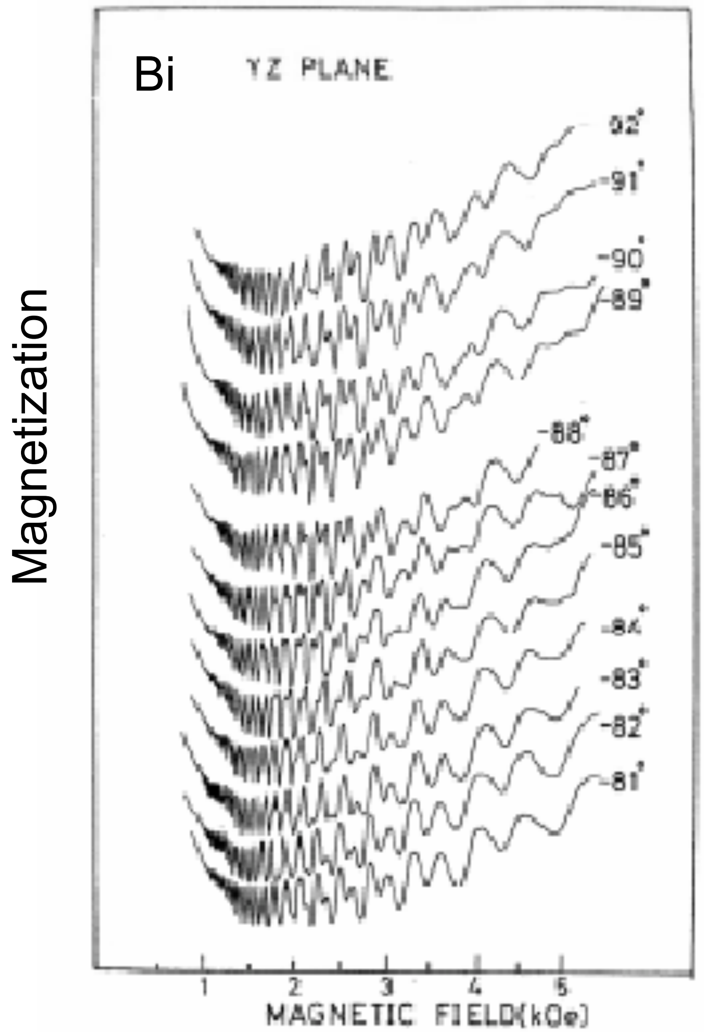


If we are able to measure T (or frequency), we will obtain information about the shape of Fermi surface

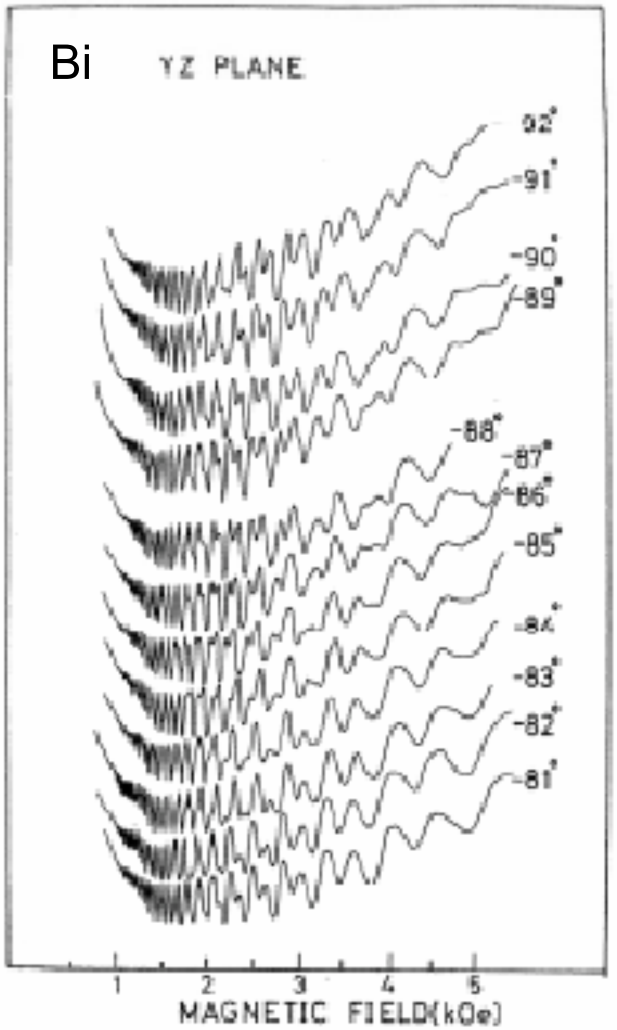
Discovered 1930:
de Haas – van Alphen Oscillations

W.J. de Haas
(1878-1960)

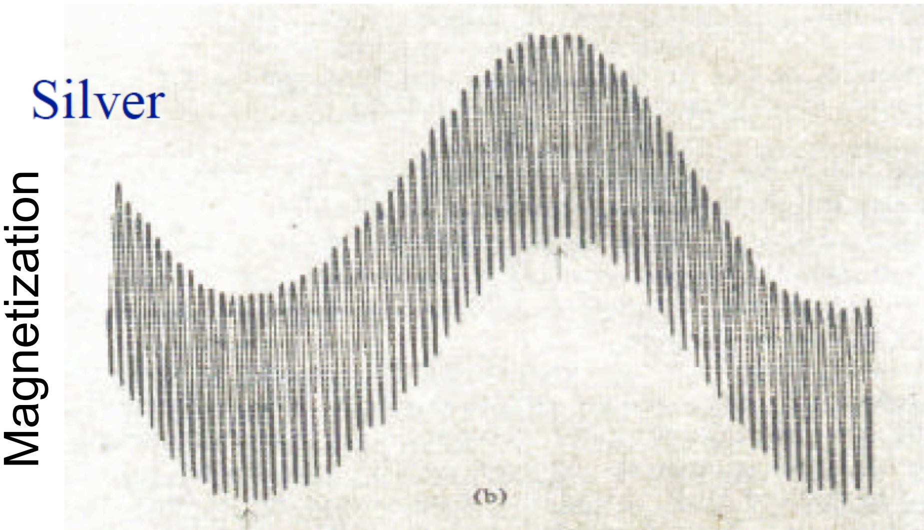
P.M. van Alphen
(1906-1967)



Magnetization



Magnetization



1/H

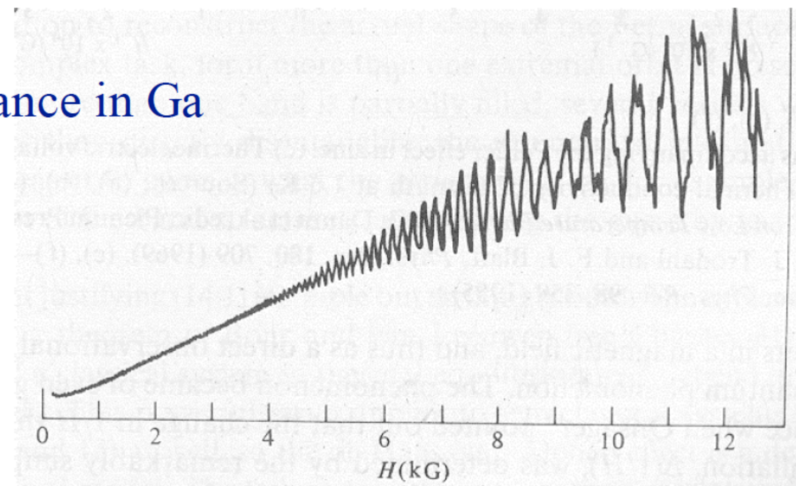
T (or frequency) depends on 1/H

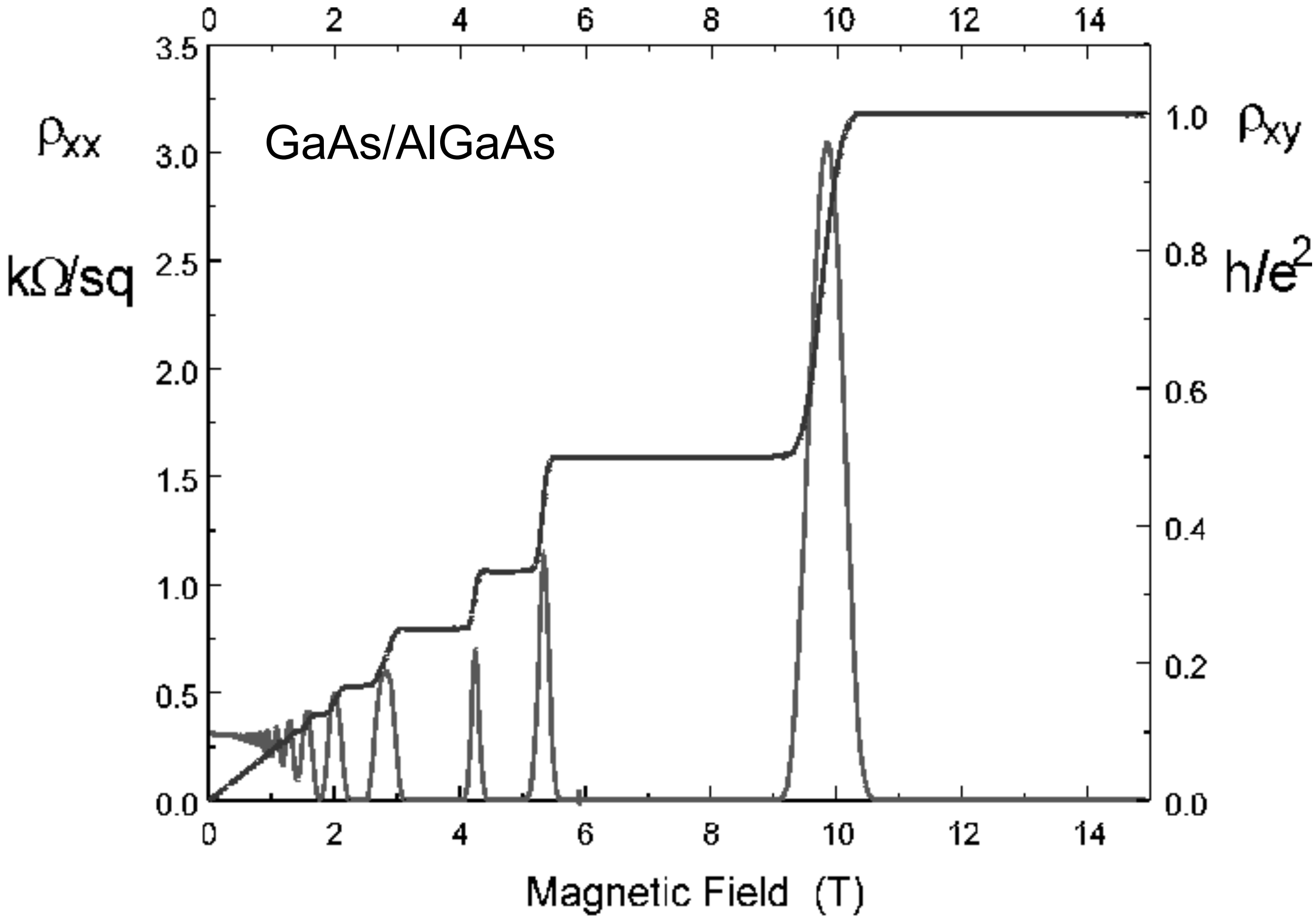
Discovered 1930: Shubnikov-de Haas Oscillations

Lev Shubnikov
(1901-1937)

W.J. de Haas
(1878-1960)

Resistance in Ga

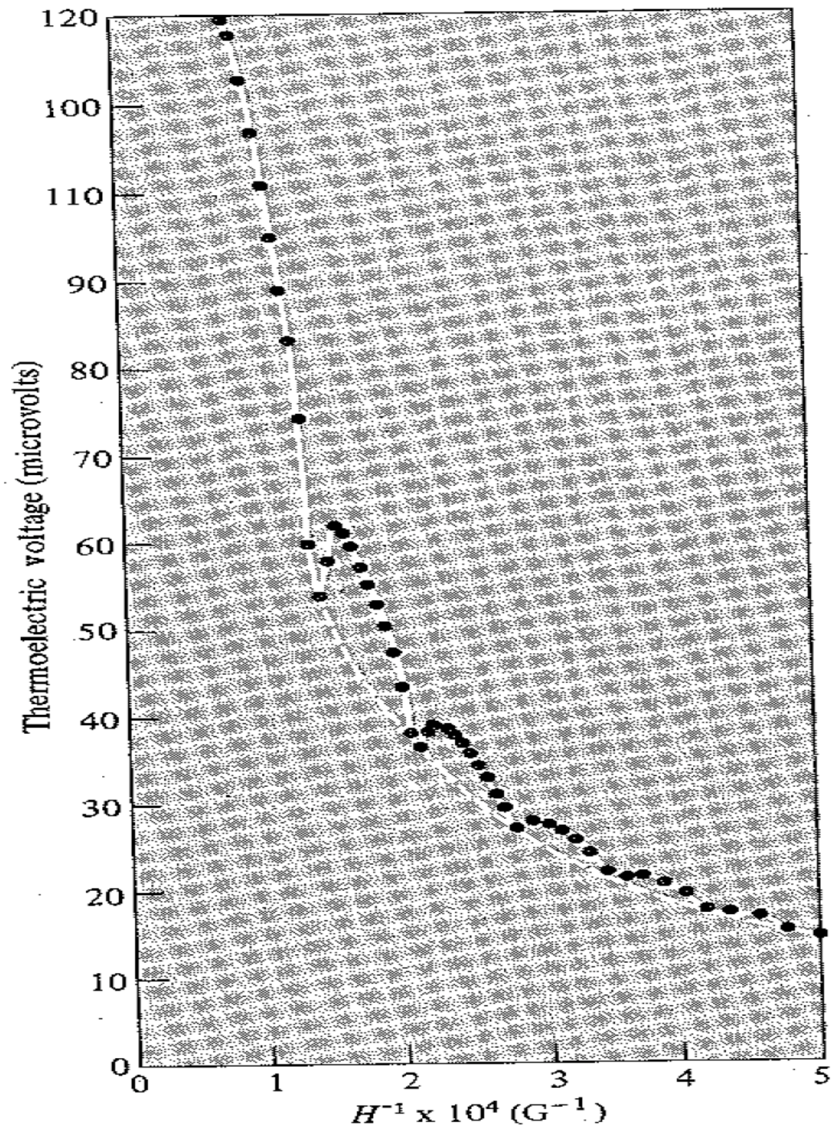


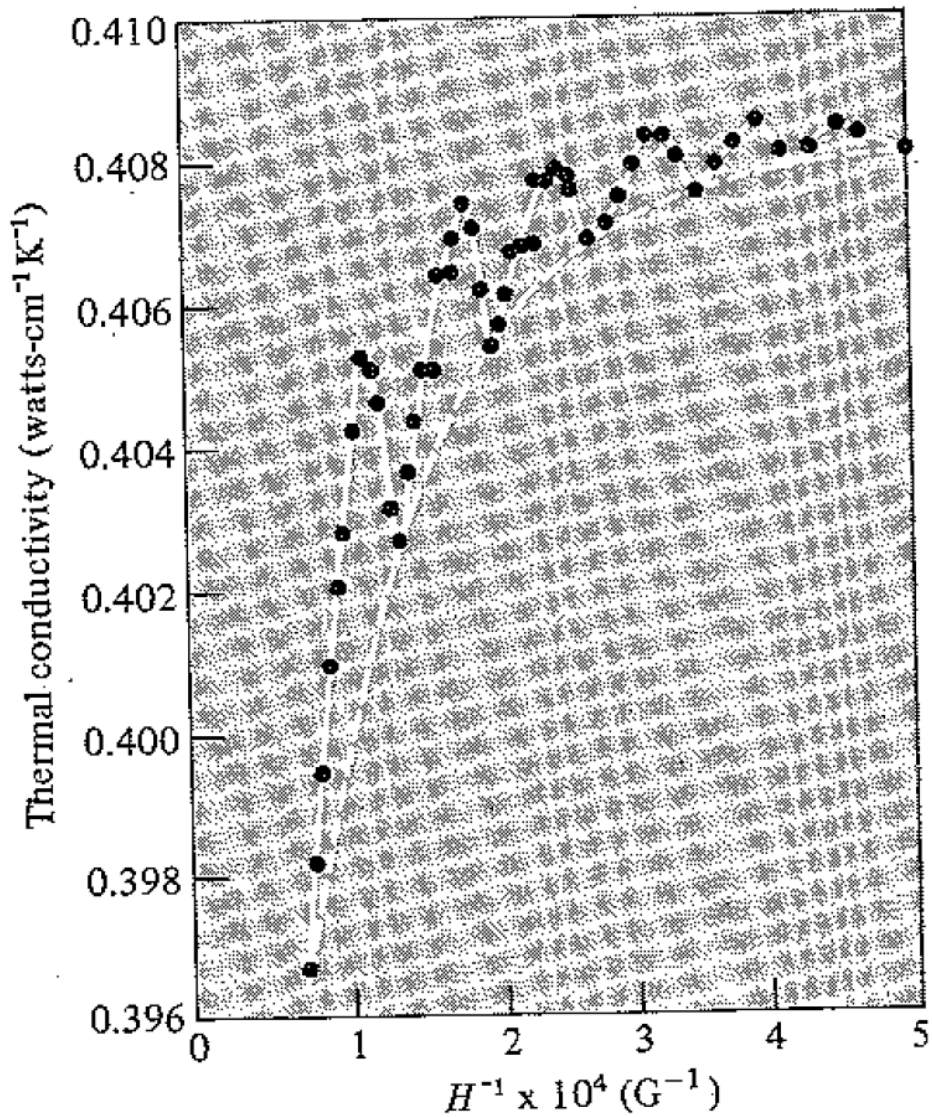


Klaus von Klitzing

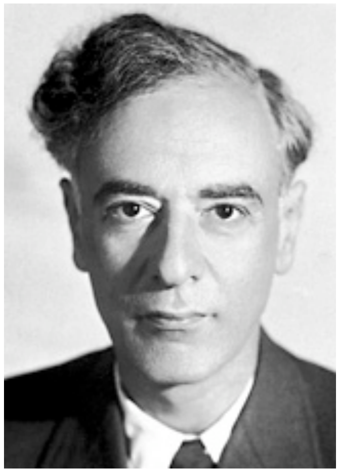
Nobel prize 1985

Similar oscillations are also observed in other Physical quantities…





What is the Origin of the Oscillation ?



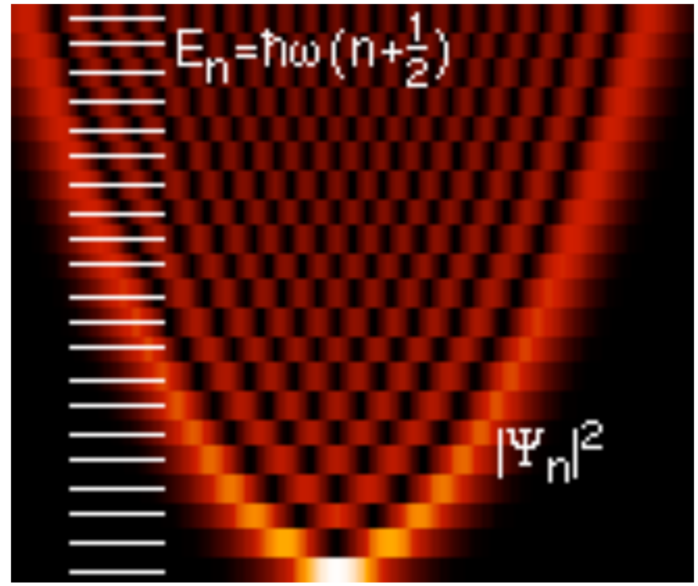
Lev Landau

Nobel prize 1962

Landau found that the cyclotron orbits are quantized in magnetic fields

$$\varepsilon_n^H = \hbar\omega_c\left(n + \frac{1}{2}\right)$$

$$\omega_c = \frac{eH}{mc}$$



Landau Levels

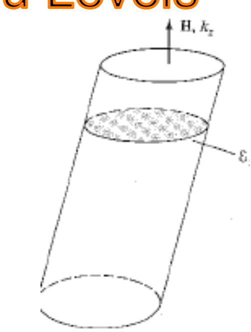
At each Landau Level, the maximum number of particles:

$$N_{LL}^M = Z(2S+1)\frac{\Phi}{\Phi_0}$$

# of electrons

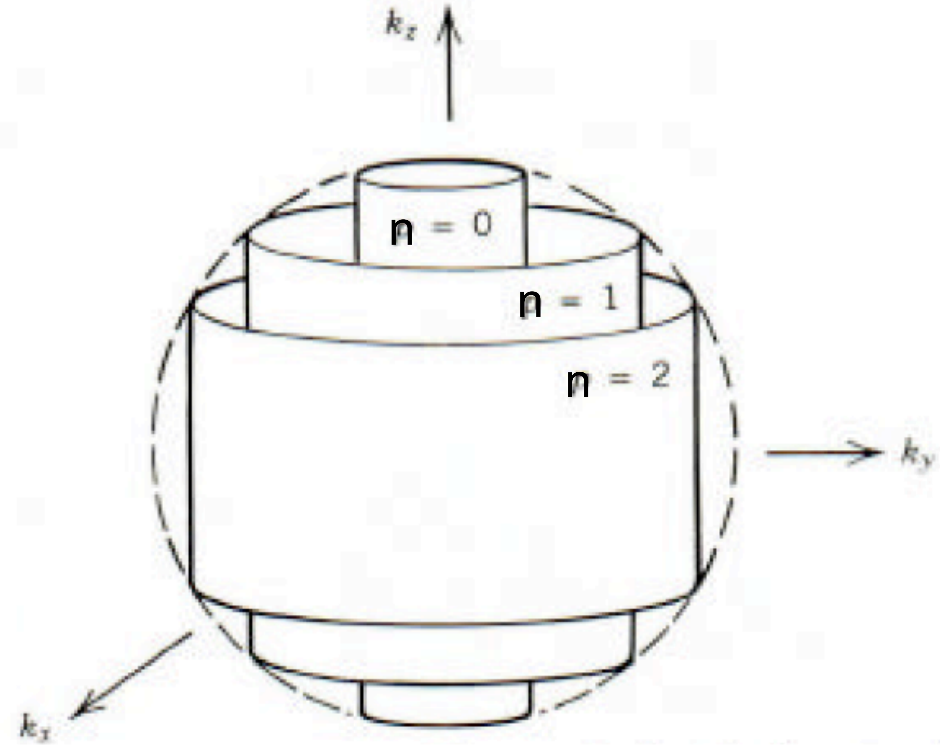
$$\Phi = HA$$

$$\Phi_0 = \frac{hc}{e} \leftarrow \text{Flux quantum}$$



Along the direction of applied magnetic field (z-direction):

$$E_{n,k_z} = \left(n + \frac{1}{2}\right)\hbar\omega_c + \frac{\hbar^2 k_z^2}{2m^*}$$



- In the presence of H, the Fermi sphere becomes a stack of cylinders (Landau tubes) If E_n ~ 0.1meV, E_F ~ 1eV $\Rightarrow$ n ~ 10000
- The radius is proportional to H

$$E_{n,k_z} = \left(n + \frac{1}{2}\right)\hbar\omega_c + \frac{\hbar^2 k_z^2}{2m^*}$$

$$E_{n+1}(k_z) - E_n(k_z) = \hbar\omega_c = \frac{h}{T(E_n(k_z), k_z)}$$

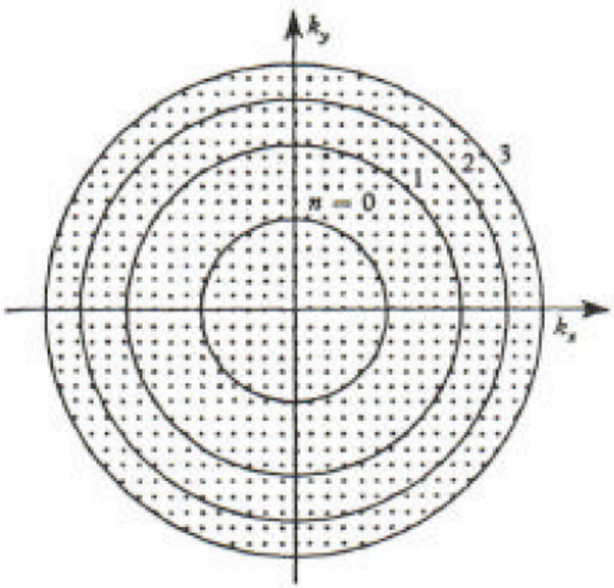
$$T(\varepsilon, k_z) = \frac{\hbar^2 c}{eH}\frac{\Delta A(\varepsilon, k_z)}{\Delta\varepsilon}$$

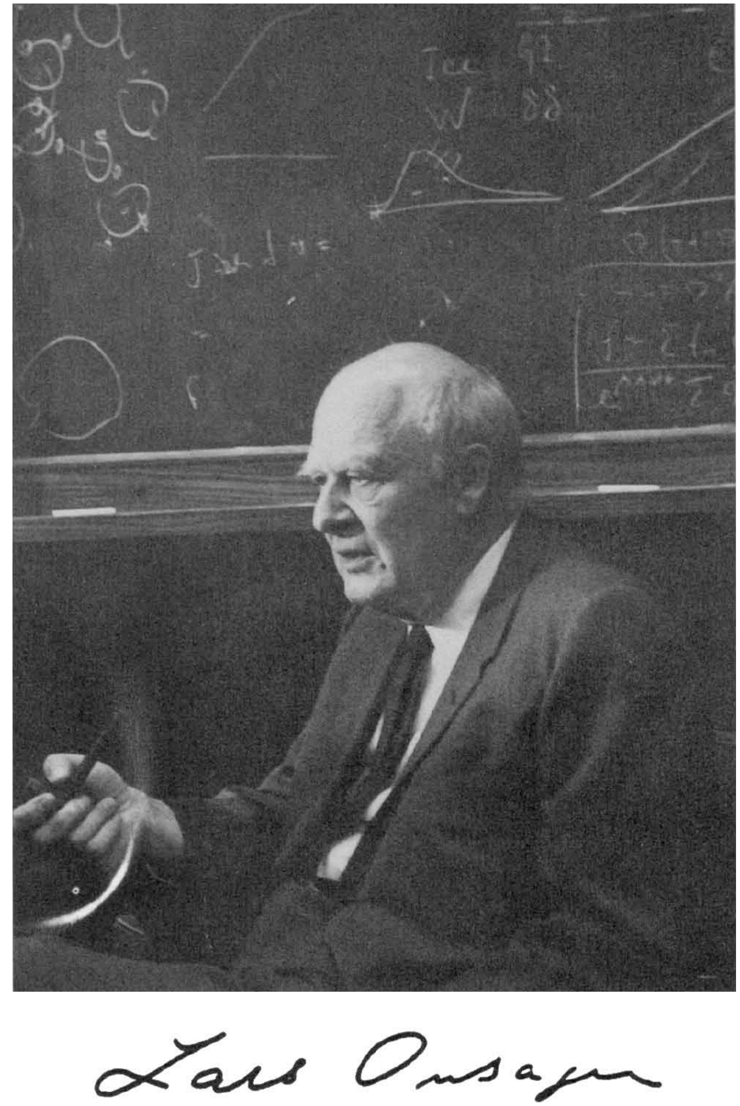
$$\boxed{\Delta A = \frac{H}{\Phi_0}}$$

- The area change depends on H

In k-orbit:

$$A_n^k = \left(n + \frac{1}{2}\right)\frac{H}{\Phi_0}$$





Nobel Prize in Chemistry
1968

$$A(E_{n+1}) - A(E_n) = \Delta A = \frac{H}{\Phi_0}$$

$$A_n = \left(n + \frac{1}{2}\right)\frac{H}{\Phi_0} \qquad A_{n+1} = \left((n+1) + \frac{1}{2}\right)\frac{H'}{\Phi_0}$$

$$\frac{A_{n+1}}{H'} - \frac{A_n}{H} = \frac{1}{\Phi_0}$$

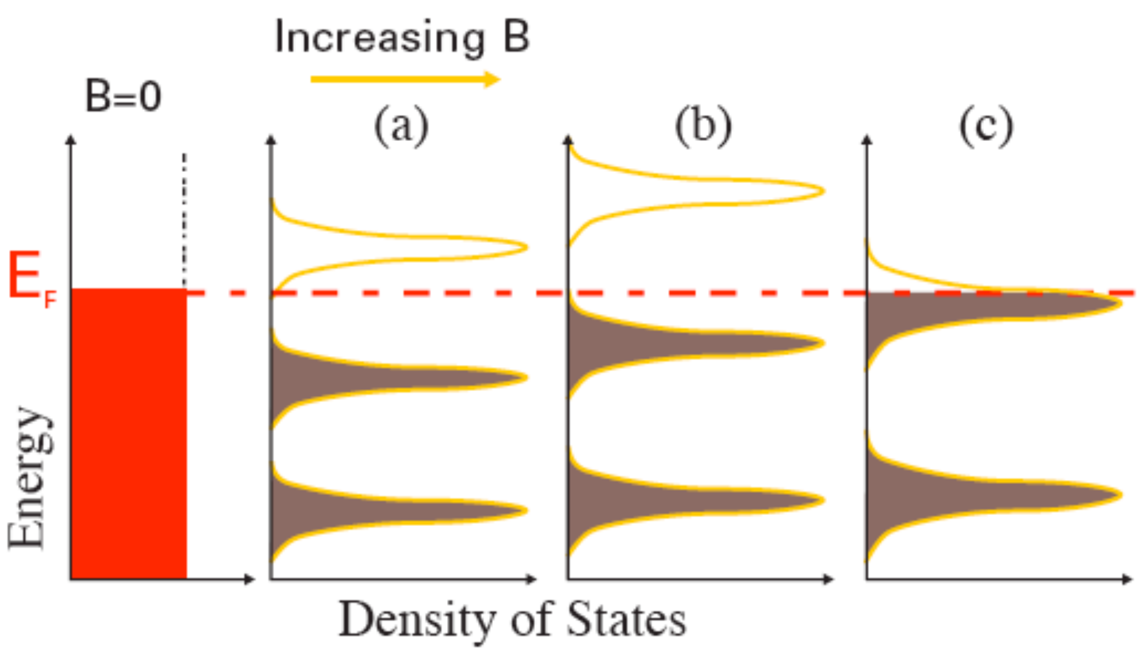
$$A\left(\frac{1}{H'} - \frac{1}{H}\right) = A\Delta\left(\frac{1}{H}\right) = \frac{1}{\Phi_0}$$

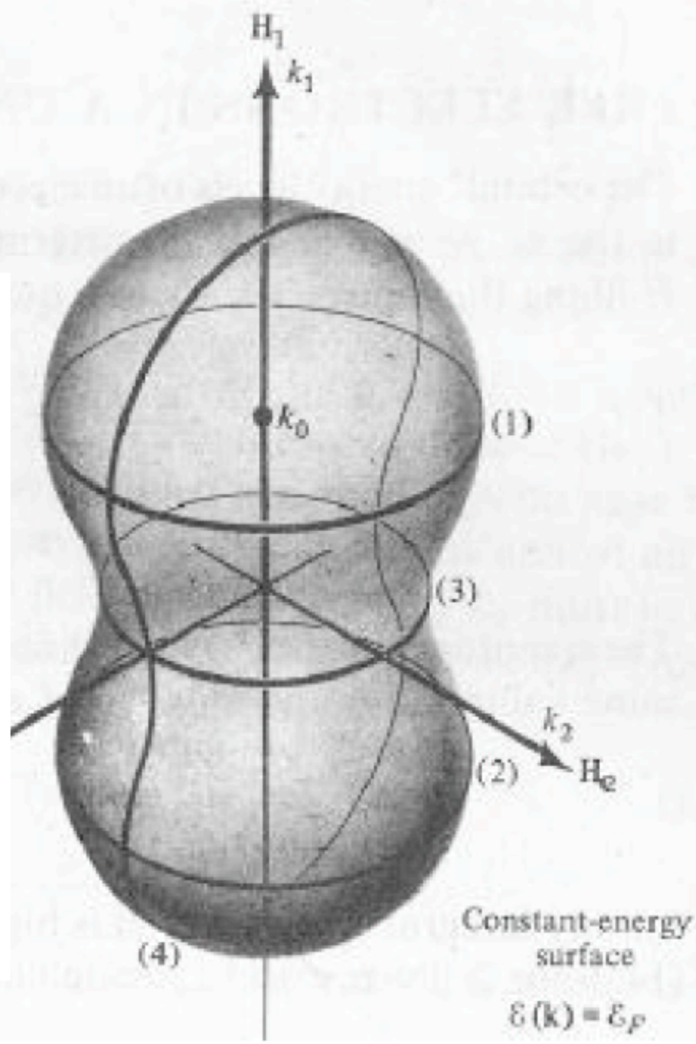
$$\Delta\left(\frac{1}{H}\right) = \frac{1}{\Phi_0}\frac{1}{A}$$

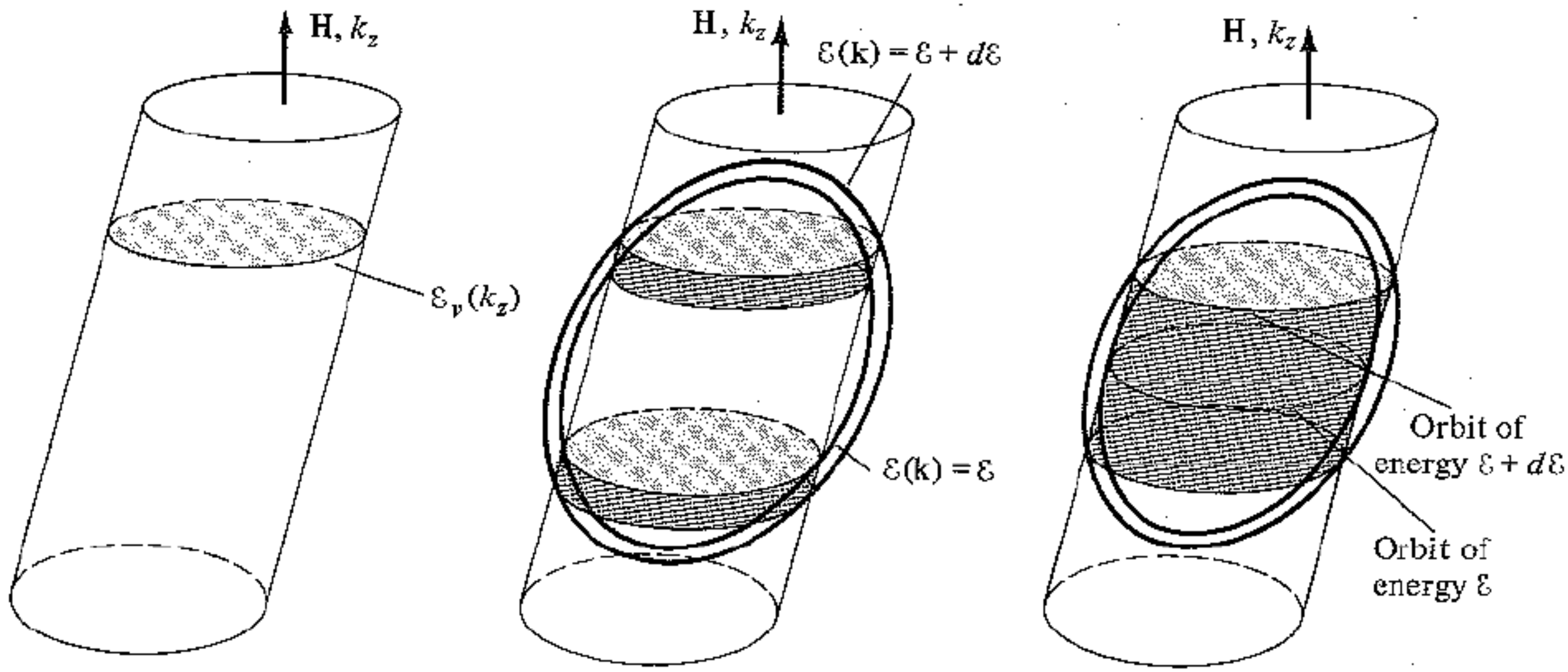
$$\Delta\left(\frac{1}{H}\right) = \frac{1}{\Phi_0}\frac{1}{A}$$

$$A = A_{extreme} = \Phi_0 / \Delta\left(\frac{1}{H}\right)$$

A peak in the Density-of-states







w/o extremal orbit w/ extremal orbit

Determination of the Fermi Surface

1. Measure physical quantities, such as magnetization and/or magnetoresistance;

2. In order to observe quantum oscillations in these quantities, $\hbar\omega_c >> k_B T$

Require:
- high field to increase ω_c
- low T

3. Determination of frequencies in 1/H

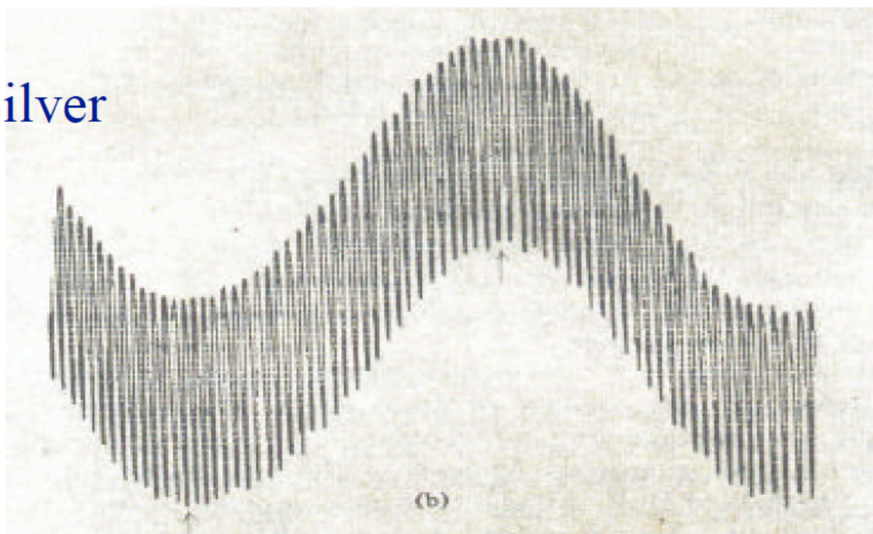
4. Determination of extreme areas via $A_e = \Phi_0 / \Delta\left(\frac{1}{H}\right)$

Example I: Silver

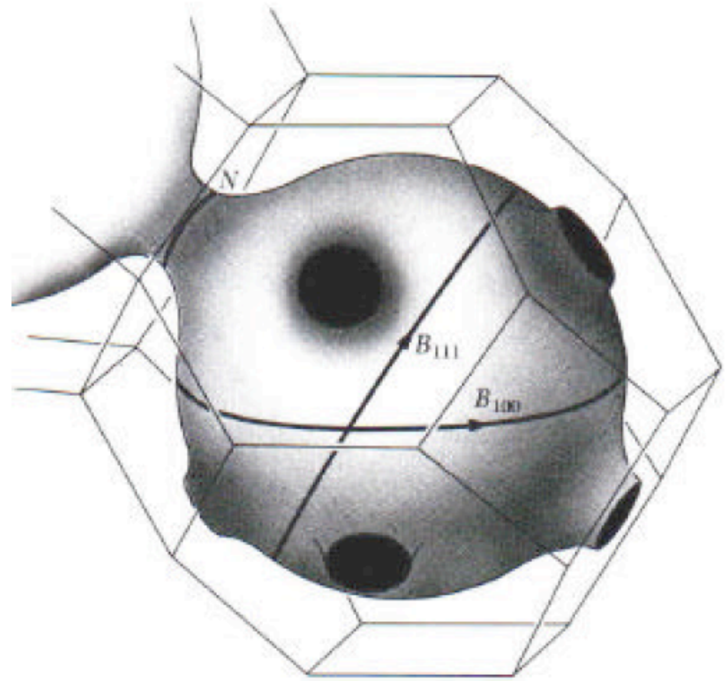
In the dHvA experiment of silver, the two different periods of oscillation are due two different extremal orbits

Silver

Magnetization

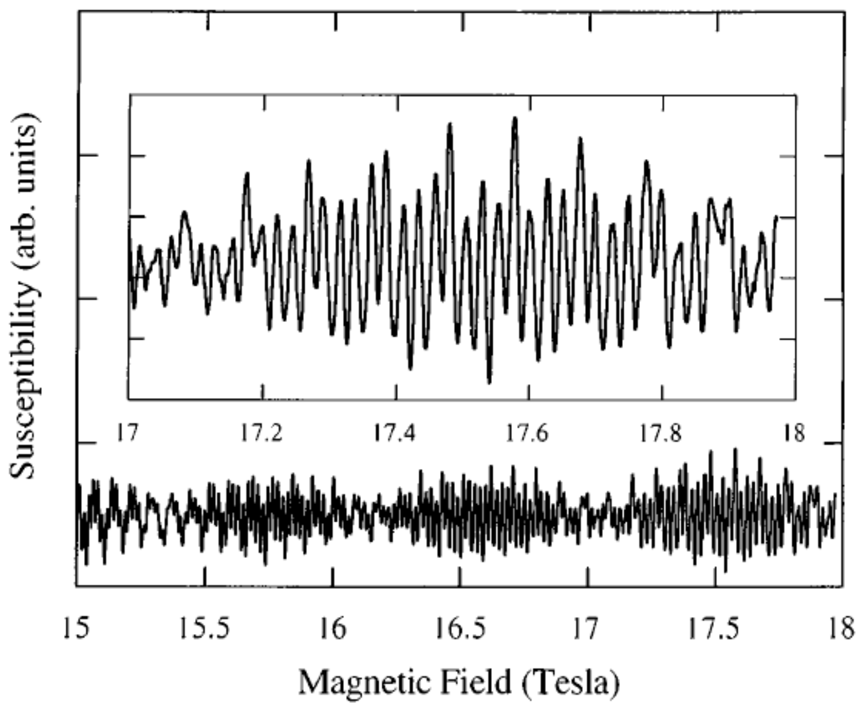


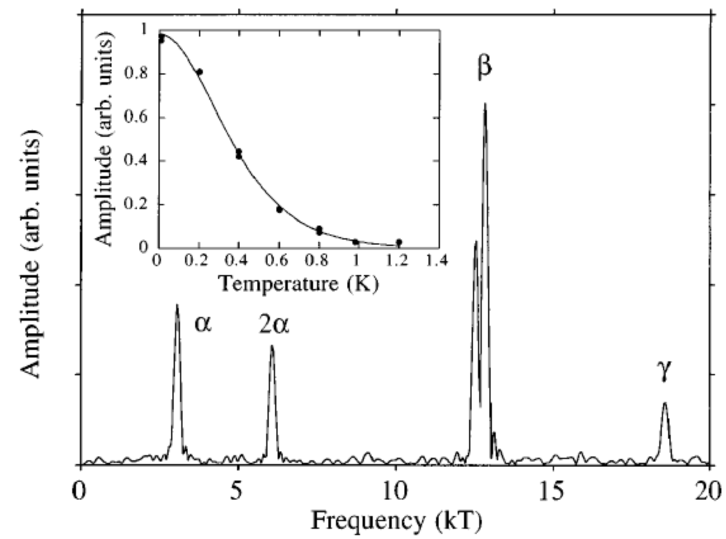
1/H

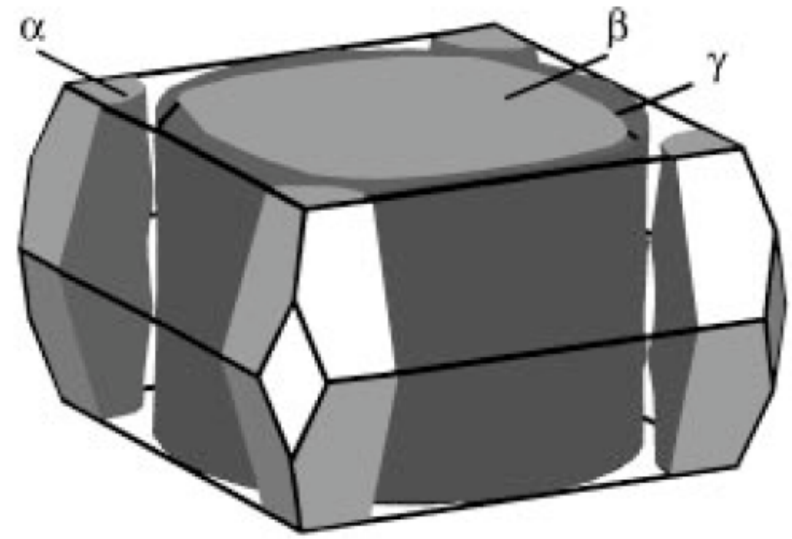


⇒ These allow us to determine the "neck" and "belly"

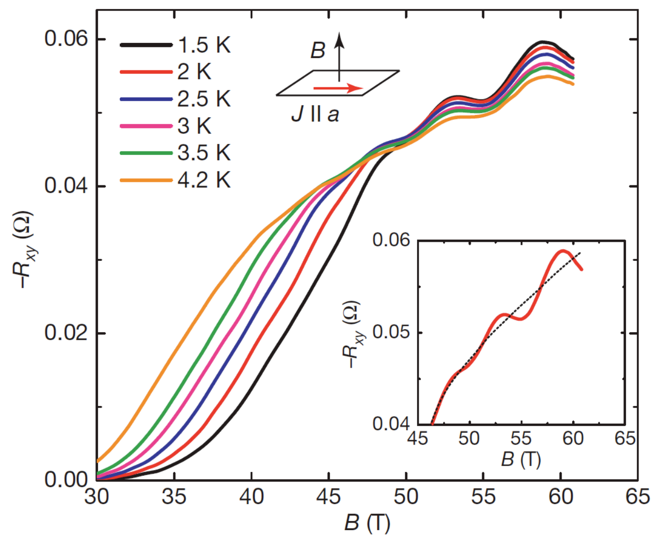
Example II: Sr_2RuO_4



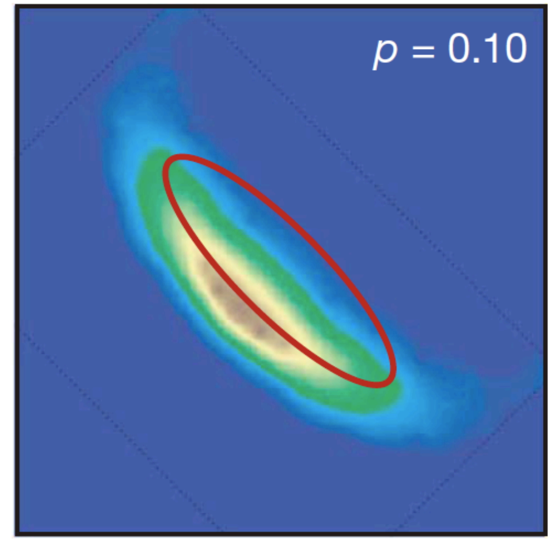




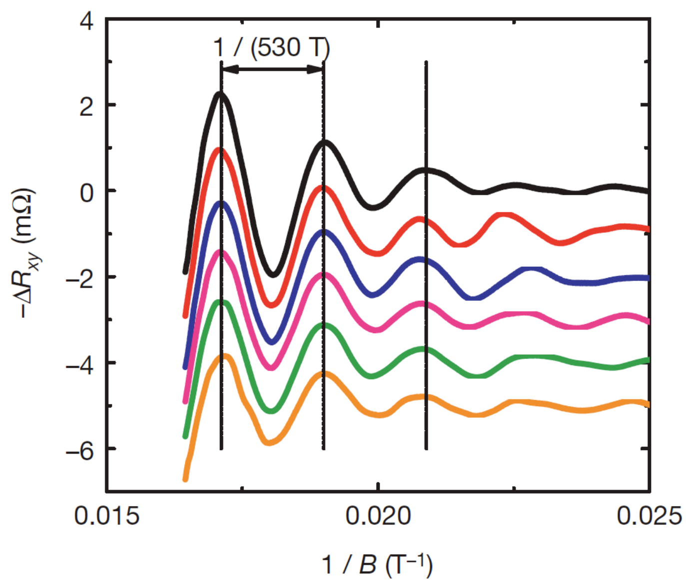
$YBa_2Cu_3O_{6+y}$



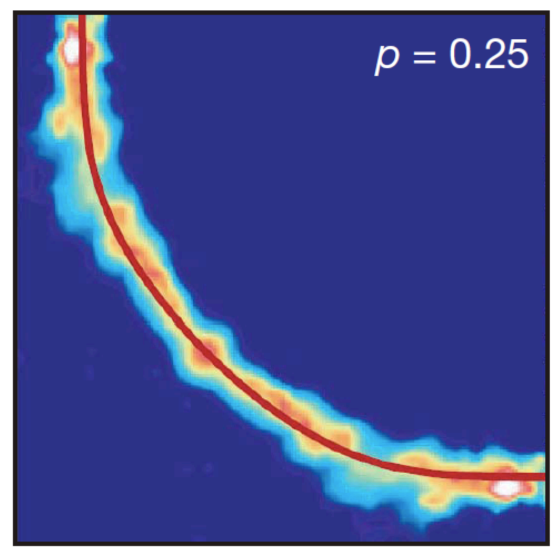
0.06
0.04
0.02
0.00
−R_{xy} (Ω)
1.5 K
2 K
2.5 K
3 K
3.5 K
4.2 K
B
J ‖ a
30 35 40 45 50 55 60 65
B (T)
0.06
0.05
0.04
−R_{xy} (Ω)
45 50 55 60 65
B (T)



p = 0.10



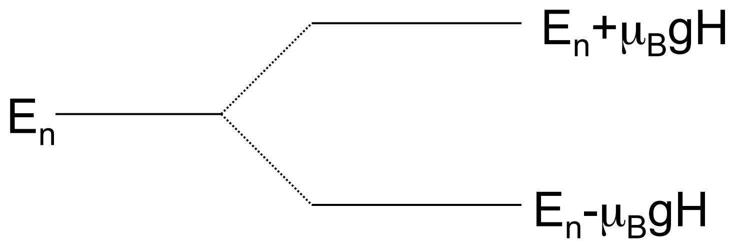
1 / (530 T)
4
2
0
−2
−4
−6
−ΔR_{xy} (mΩ)
0.015 0.020 0.025
1 / B (T−1)



p = 0.25

Effects of spin-orbit coupling:

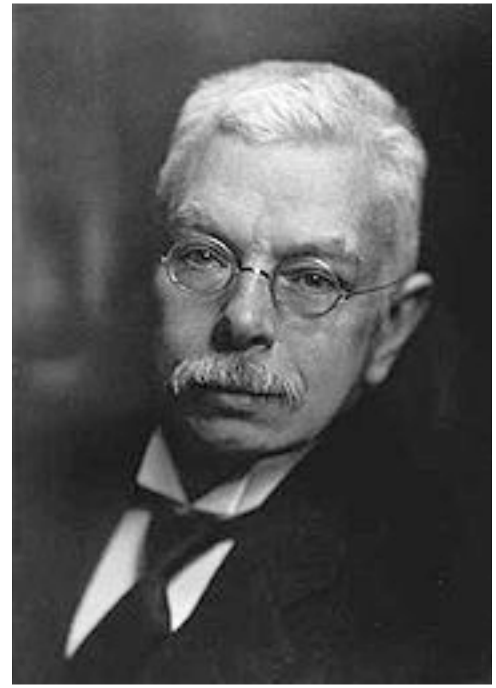
Zeeman Splitting



$$g(\varepsilon)=\frac{1}{2}g_0\left(\varepsilon+\mu_B gH\right)+\frac{1}{2}g_0\left(\varepsilon-\mu_B gH\right)$$

$$2\mu_B gH=\hbar\omega_c$$

Cancellation of oscillations



Pieter Zeeman
(1965-1943)

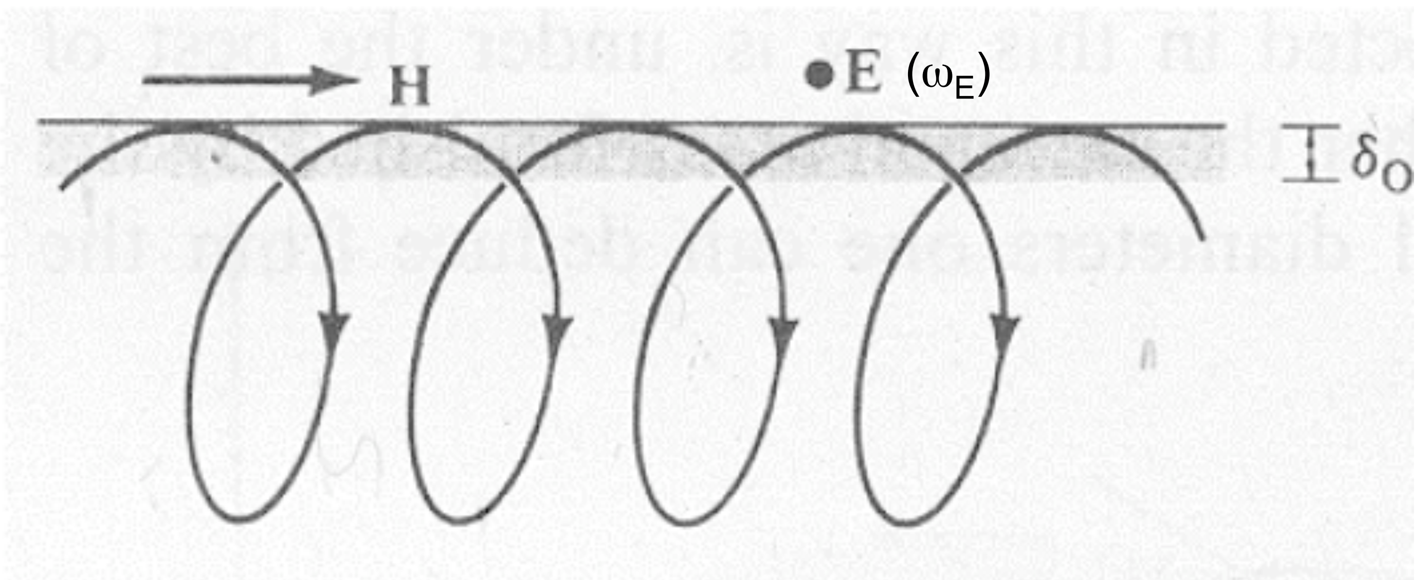
Nobel Prize in 1902

Other Fermi Surface Probes:

Azbel-Kaner cyclotron resonance (1956)

= a steady magnetic field to make cyclotron motion

+ an oscillating electric field to induce resonance



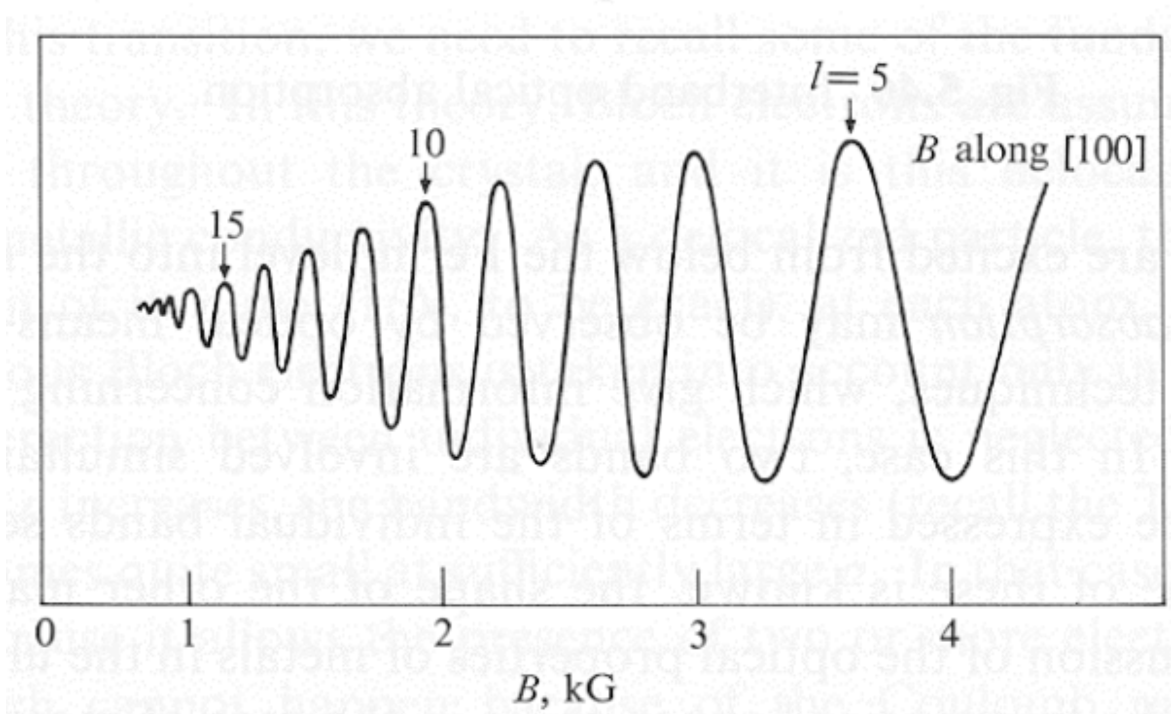
Cyclotron Resonance:

$$\omega_E = \hbar\omega_c$$

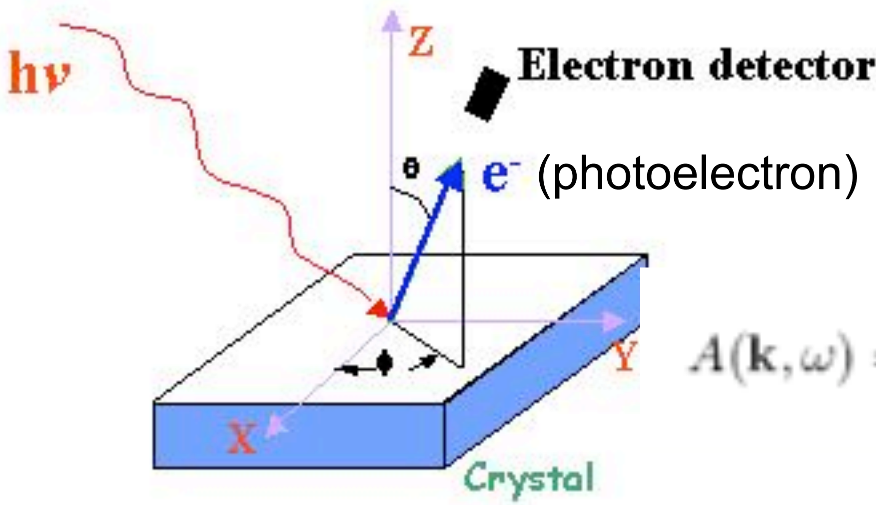
Electrons can absorb energy from the field

Cyclotron resonance

$$\frac{1}{H} = \frac{2\pi e}{\hbar^2 c\omega_E}\frac{1}{\partial A/\partial\varepsilon}n$$



Angle-Resolved Photoemission Spectroscopy



$$I(\mathbf{k},\omega) = I_0(\mathbf{k},\nu,\mathbf{A})f(\omega)A(\mathbf{k},\omega)$$

$$A(\mathbf{k},\omega) = -\frac{1}{\pi}\frac{\Sigma''(\mathbf{k},\omega)}{[\omega-\epsilon_{\mathbf{k}}-\Sigma'(\mathbf{k},\omega)]^2+[\Sigma''(\mathbf{k},\omega)]^2}$$

?

$$I \sim A(k,\omega).f(\omega)$$

$$I \propto f(\omega)$$

indicating that ARPES measures the occupied states

Energy Conservation

$E_B = h\nu - E_{kin} - \Phi$

Momentum Conservation

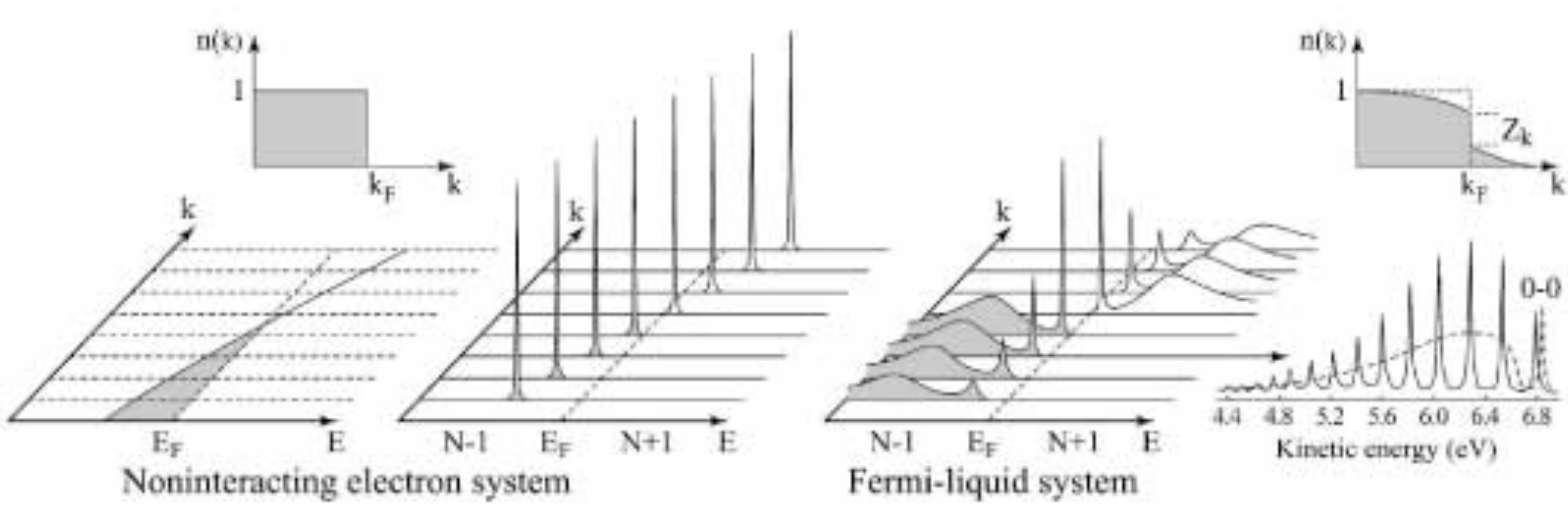
$K_{||} = k_{||} + G_{||}$

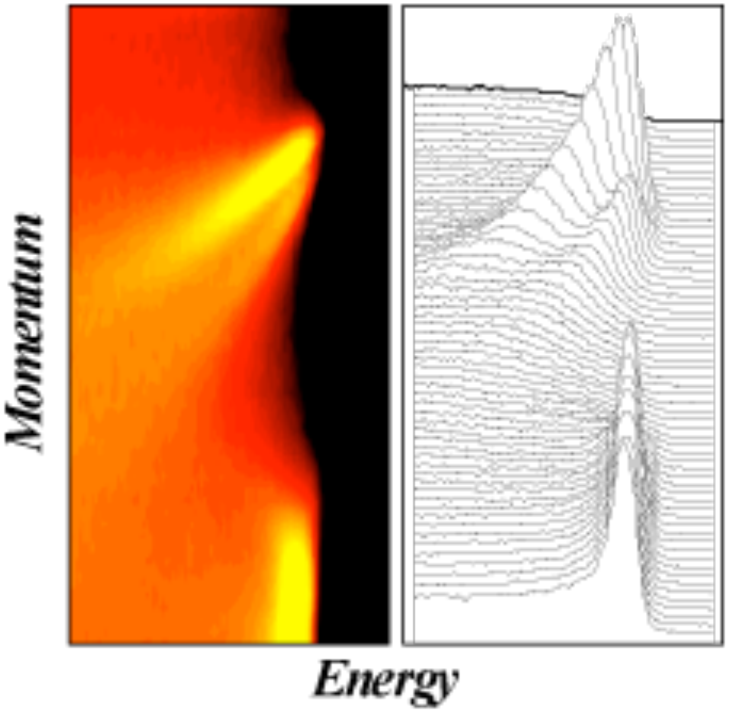
Work function of the analyzer

Kinetic energy of the outgoing electron

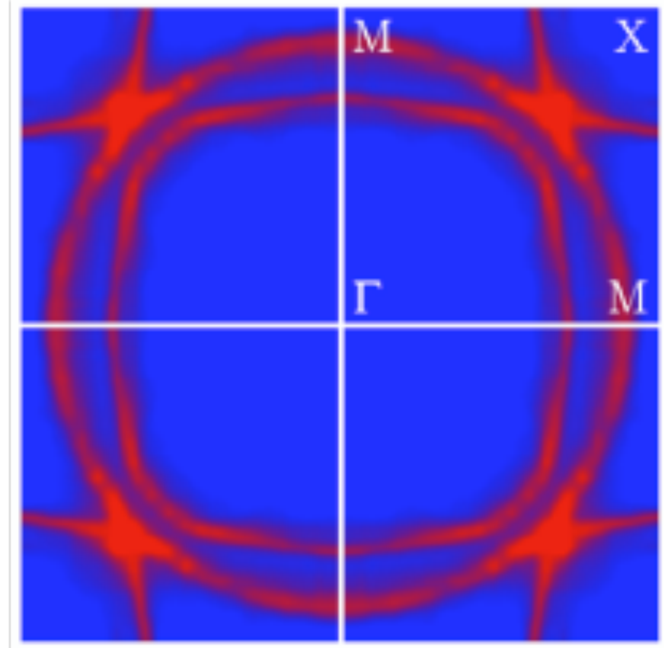
⇒ Gives information about distribution of electronic states

Energy-momentum dependence

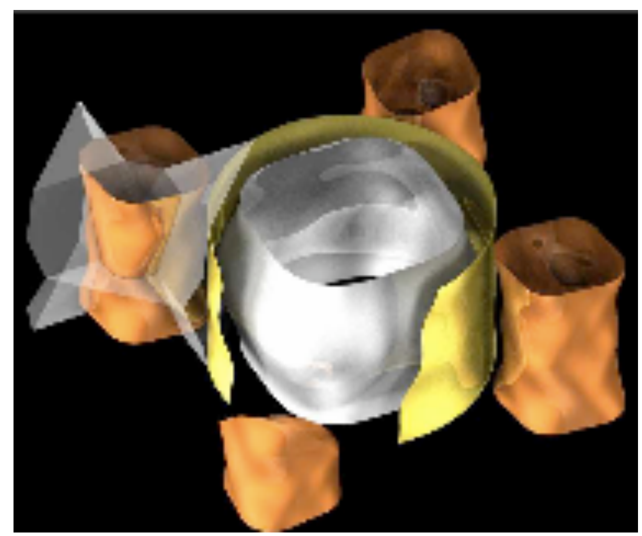




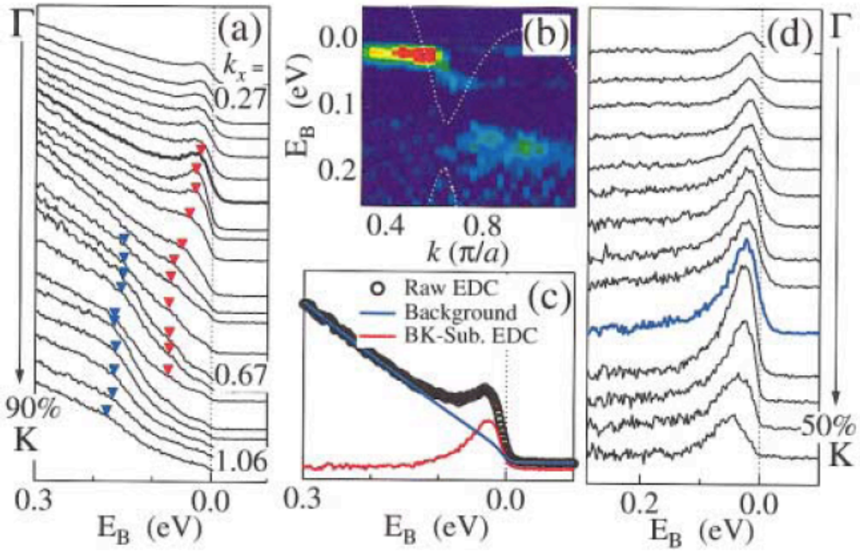
2D



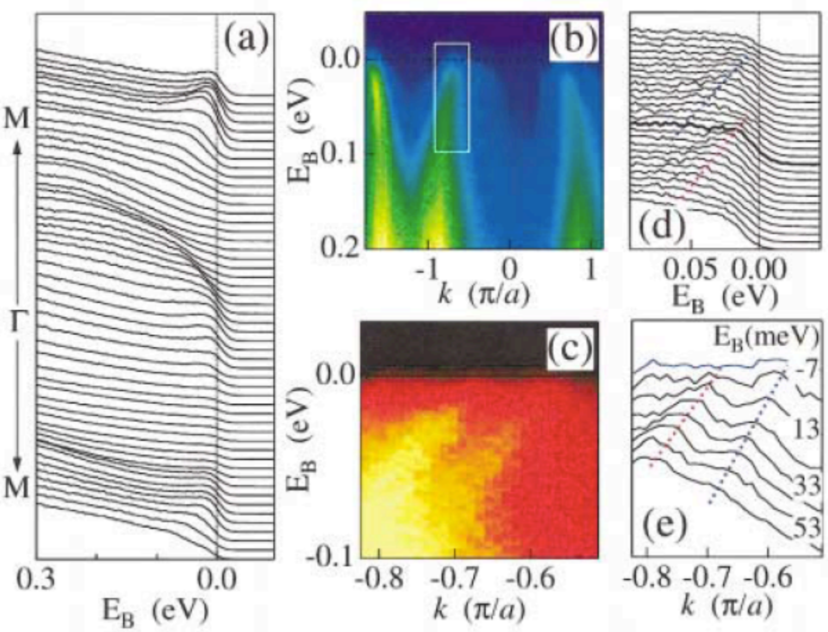
3D

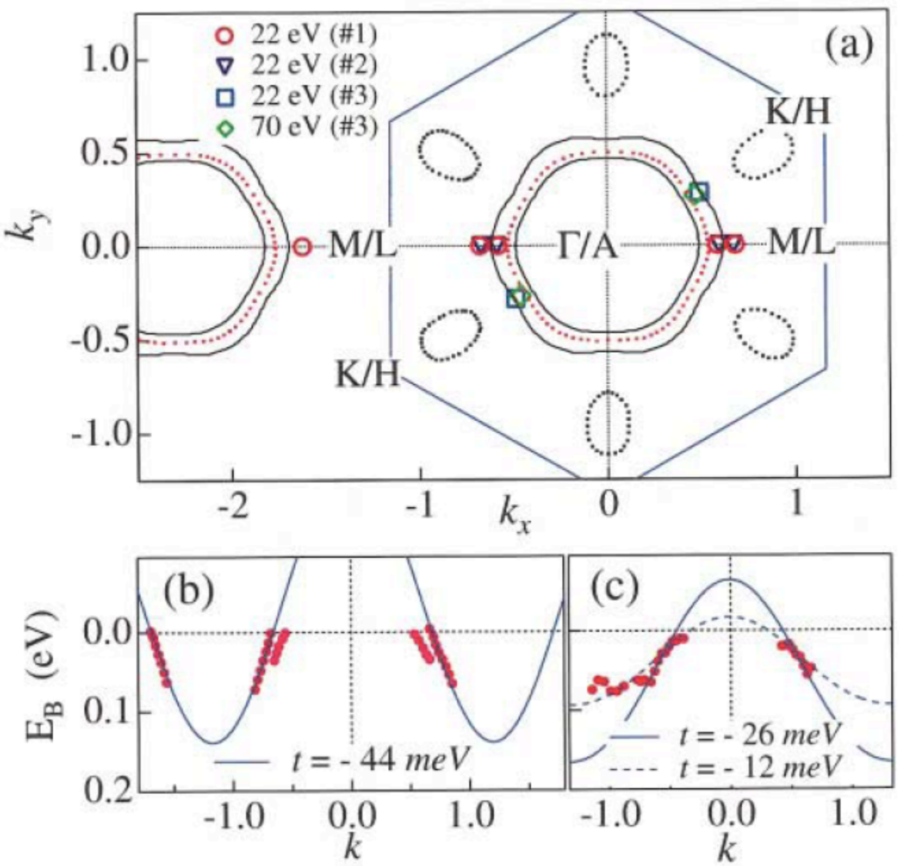


Sr_2RuO_4


(a)
Γ
K
90%
0.27
0.67
1.06
k_x =
E_B (eV)
(b)
k (π/a)
(c)
Raw EDC
Background
BK-Sub. EDC
(d)
50%

$Na_{0.6}CoO_2$


(a)
M
Γ
M
E_B (eV)
(b)
k (π/a)
(c)
(d)
(e)
E_B(meV)
-7
13
33
53


(a)
22 eV (#1)
22 eV (#2)
22 eV (#3)
70 eV (#3)
K/H
M/L
Γ/A
k_y
k_x
(b)
(c)
E_B (eV)
k
t = - 44 meV
t = - 26 meV
t = - 12 meV

Homework (due on 11/4/2010)

Explain how to measure Fermi surface using STM according to the article [Science **323**, 1190 (2009)]

1190 27 FEBRUARY 2009 VOL 323 SCIENCE

Seeing the Fermi Surface in Real Space by Nanoscale Electron Focusing

Alexander Weismann,[1,2] Martin Wenderoth,[1*] Samir Lounis,[3] Peter Zahn,[4] Norbert Quaas,[1] Rainer G. Ulbrich,[1] Peter H. Dederichs,[3] Stefan Blügel[3]