Announcements:

Mid-Term Exam:

Time: 9:00 am – 10:30 am
Thursday (Oct. 14th)

Location: 106 Nicholson Hall

Style: open book

Phonons in Metals

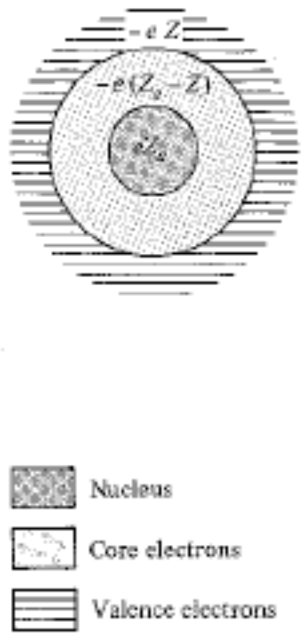
Recall:

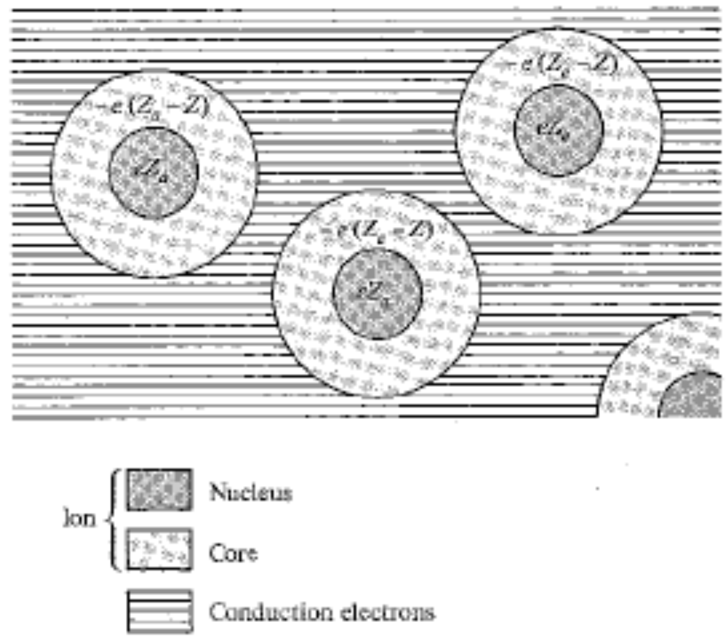
$$C_v = C_{electron} + C_{phonon}$$

$$\kappa = \kappa_{electron} + \kappa_{phonon}$$

$$\rho = \rho_{electron} + \rho_{phonon}$$

In Metals





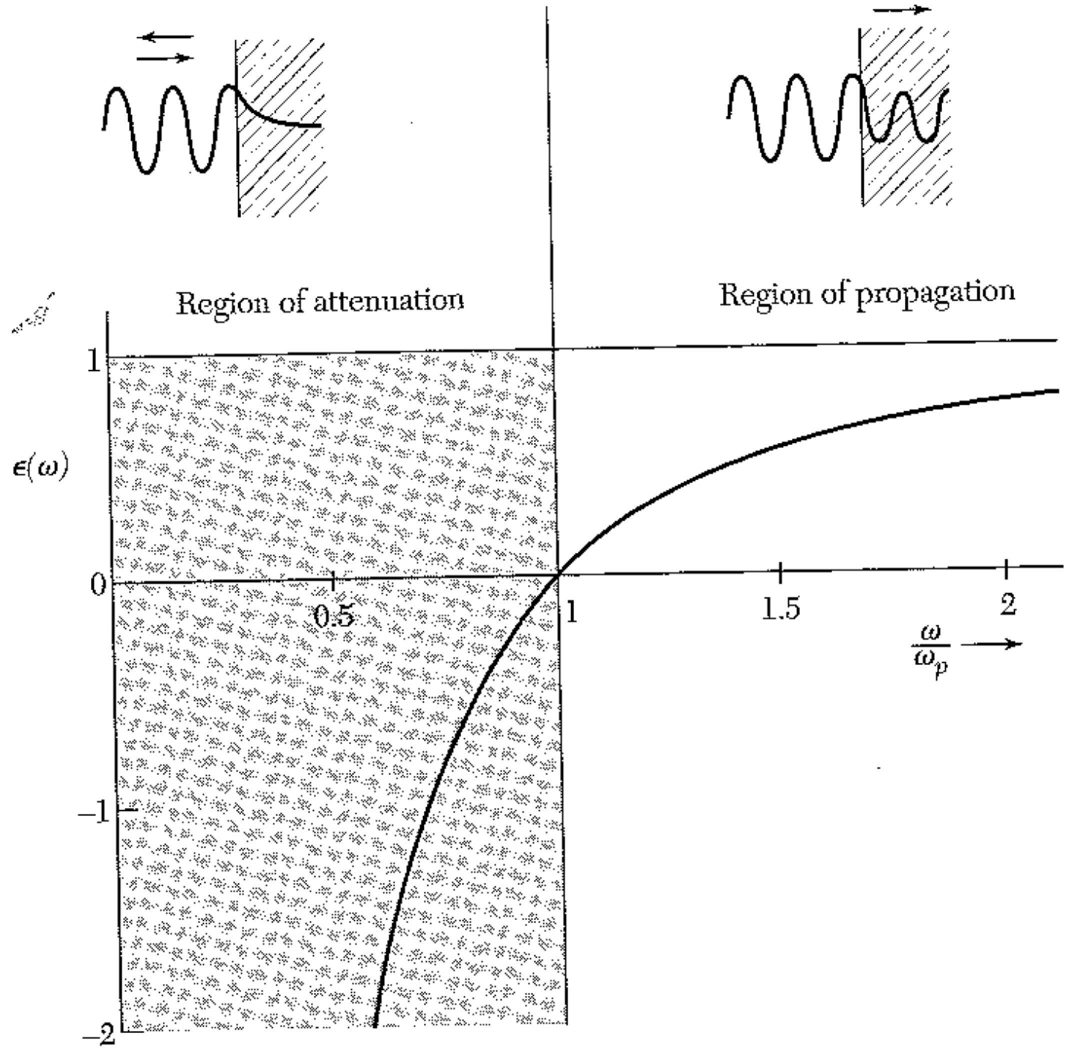
- ✪ ions are charged
- ✪ conduction electrons are present

Recall:

$$\omega_p^2 = \frac{4\pi n_e e^2}{m}$$

	Name	Field
	Electron	—
	Photon	Electromagnetic wave
	Phonon	Elastic wave
	Plasmon	Collective electron wave
	Magnon	Magnetization wave
–	Polaron	Electron + elastic deformation
–	Exciton	Polarization wave

$$\varepsilon(\omega) = 1 - \frac{\omega_p^2}{\omega^2} \qquad \varepsilon(\omega) < 0 \qquad \varepsilon(\omega) > 0$$



Electrons

e

m

n_e

$$\omega_p^2 = \frac{4\pi n_e e^2}{m}$$

electronic plasma frequency

ions

Ze

M

$n_{ion} = n_e / Z$

$$\Omega_p^2 = \frac{4\pi n_i (Ze)^2}{M}$$

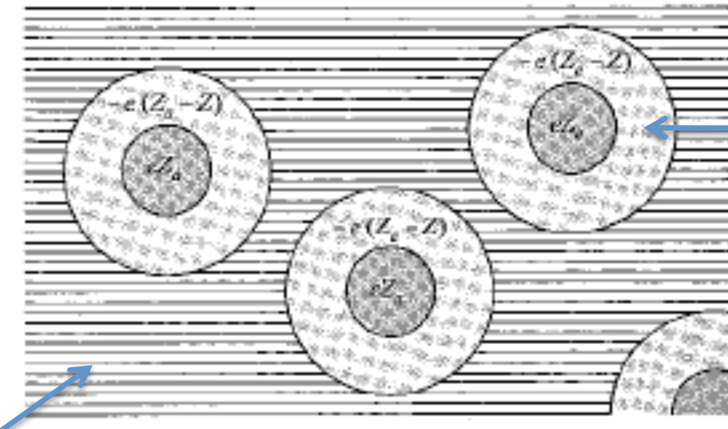
ionic plasma frequency

Phonon dispersion

$$\omega(\mathbf{k})^2 = \frac{\Omega_p^2}{\varepsilon(\mathbf{k})}$$

← electron gas dielectric constant

Electrons ions



Ions provides electrostatic potential

Electron gas is deformed by

(1) The electrostatic potential of the positive charge distribution;
(2) induced electrostatic potential of the deformation of electron gas itself

$$\varepsilon\left(0,\mathbf{k}\right)=1+\frac{k_0^2}{k^2}$$ -- dielectric constant of Thomas-Fermi form

k_0^{-1} -- Thomas-Fermi screening length

Longitudinal Plasma Oscillations

Define: $\varepsilon(\omega_L) = 0$

$$\varepsilon_{electron}(\omega,\mathbf{k}) = 1 + \frac{k_0^2}{k^2} \qquad \varepsilon_{phonon}(\omega,\mathbf{k}) = 1 - \frac{\Omega_p^2}{\omega^2}$$

$$\varepsilon(\omega_L) = \varepsilon_{electron}(\omega,k) + \varepsilon_{electron}(\omega,k) = 0$$

Ignored constant term

$$\omega^2 = \frac{\Omega_p^2}{k_0^2}k^2 = \frac{m_e}{3M_{ion}}v_F^2 k^2$$

$\omega = v_p k$

$$v_p = \sqrt{\frac{m_e}{3M_{ion}}}v_F$$

$$v_p << v_F$$

Screened Coulomb Potential

unscreened → $\varphi_0(k)$, screened → $\varphi(k)$

$$\frac{\varphi_0(k)}{\varphi(k)} = \varepsilon(0,\mathbf{k})$$

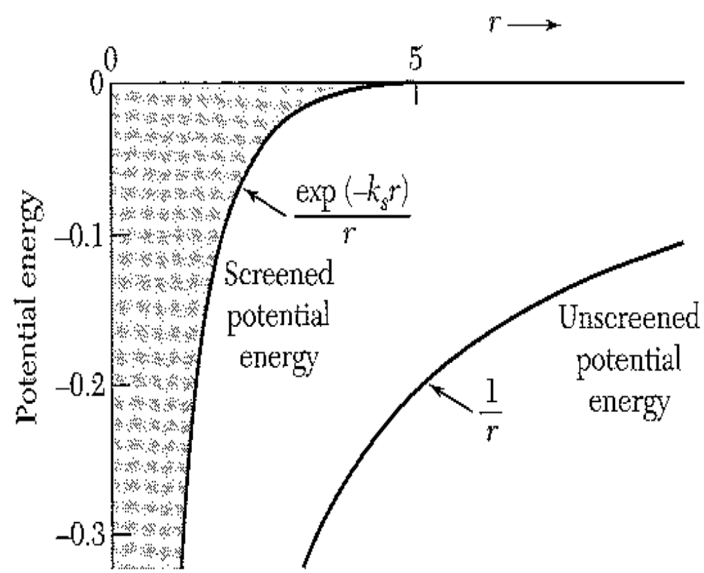
$$\varepsilon(0,\mathbf{k}) = 1 + \frac{k_0^2}{k^2}$$

$$\varphi(k) = \frac{4\pi q}{k^2 + k_0^2}$$

Fourier transform

$$\varphi(r) = \frac{4\pi q}{(2\pi)^3}\int_0^\infty dk\varphi(k)\int_{-1}^{1} d(\mathbf{cos}\,\theta)e^{(ikr\,\mathbf{cos}\,\theta)}$$

$$= \frac{q}{r}e^{(-k_0 r)} = \varphi_0(r)e^{(-k_0 r)}$$



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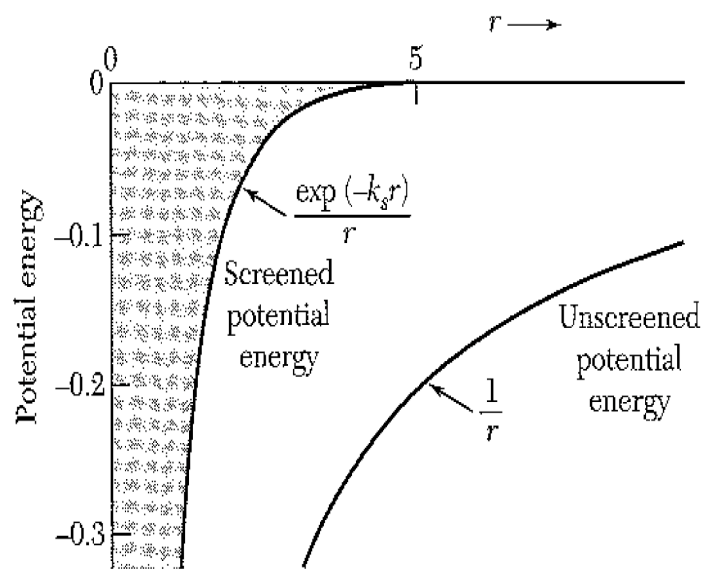
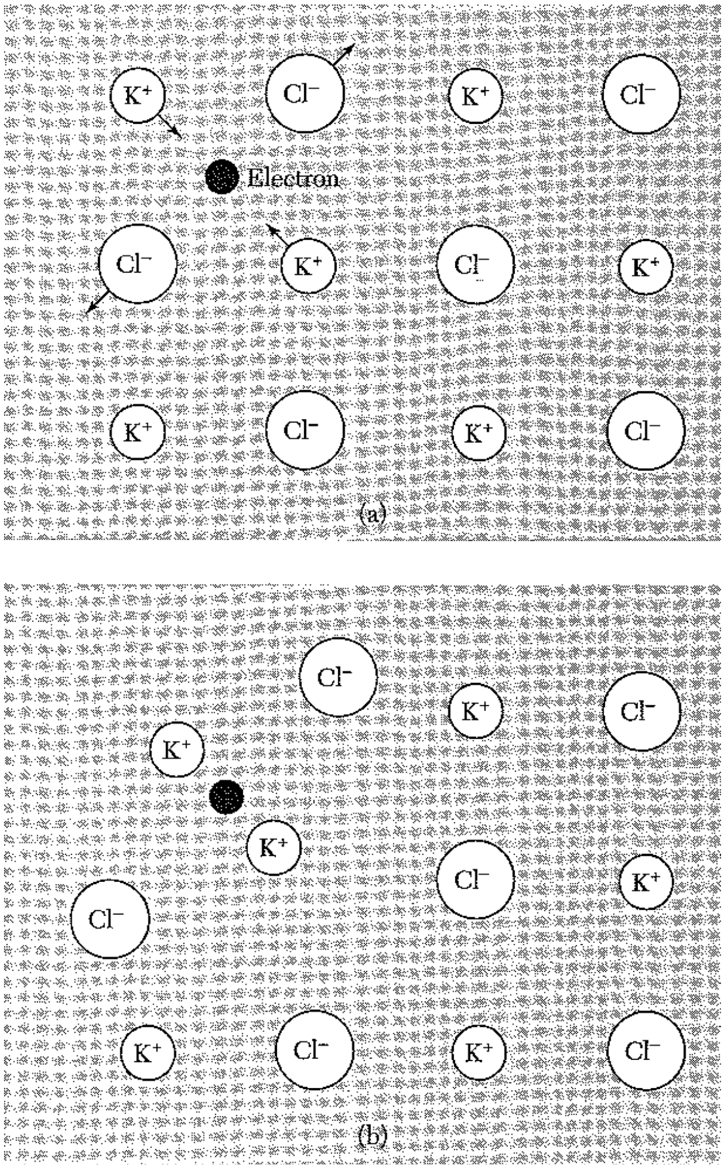


Electron-Phonon Interaction



Electron-phonon coupling constant: λ

$$\lambda = 2\frac{deformation-energy}{\hbar\omega_L}$$

Large in ionic crystals
Small in covalent crystals

Phonon Contribution to the Electronic Energy-Wave Vector Relation ε(k)

Ionic screening to
Electronic energy distribution

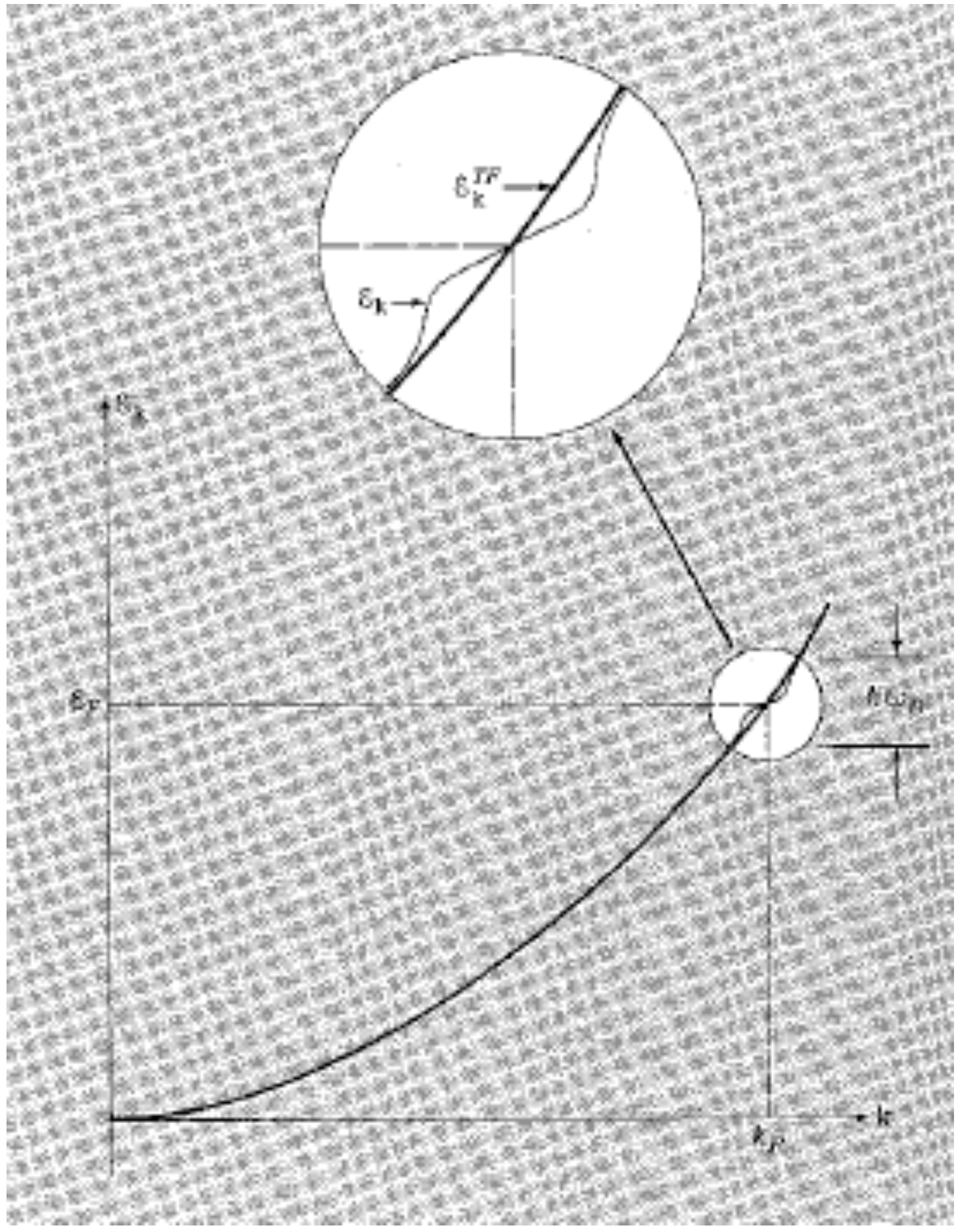
$$\varepsilon_k - \varepsilon_F = \frac{\varepsilon_k^{TF} - \varepsilon_F}{1+\lambda} \qquad (\varepsilon_k - \varepsilon_F < \hbar\omega_D)$$

$$g(\varepsilon_F) = (1+\lambda)g^0(\varepsilon_F)$$

$$\mathbf{v}(\mathbf{k}) = \frac{\mathbf{v}^0(\mathbf{k})}{1+\lambda}$$

$$\lambda = \int_{FS} \frac{dS'}{8\pi^3\hbar v(\mathbf{k}')} \frac{4\pi e^2}{(\mathbf{k}-\mathbf{k}')^2 + k_0^2}$$

Phonon Contribution to the Electronic Energy-Wave Vector Relation ε(k)



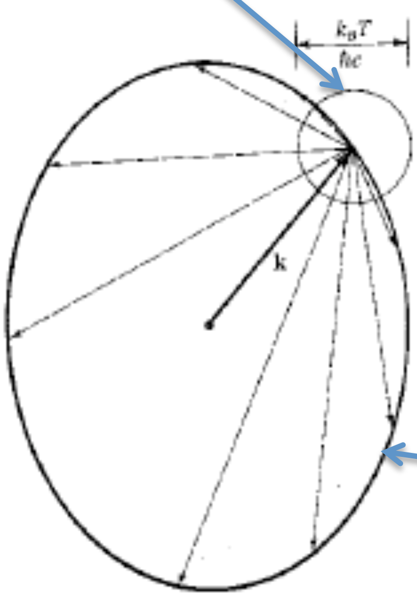
Electron-Phonon Interaction

Creation of a phonon $\qquad \varepsilon_{\mathbf{k}} = \varepsilon_{\mathbf{k}'} + \hbar\omega\left(\mathbf{k} - \mathbf{k}'\right)$

Absorption of a phonon $\qquad \varepsilon_{\mathbf{k}} = \varepsilon_{\mathbf{k}'} - \hbar\omega\left(\mathbf{k} - \mathbf{k}'\right)$

$$\omega(\mathbf{q}) = \pm\frac{1}{\hbar}\left(\varepsilon_{\mathbf{k}+\mathbf{q}} - \varepsilon_{\mathbf{k}}\right)$$

$T << \Theta_D$



$T >> \Theta_D$

Figure 26.2
Construction of the wave vectors of those phonons allowed by the conservation laws to participate in a one-phonon scattering event with an electron with wave vector **k**. Because the phonon energy is at most $\hbar\omega_D \ll \mathcal{E}_F$, the surface containing the tips of the phonon wave vectors originating from **k** differs only slightly from the Fermi surface. At temperatures well below Θ_D the only phonons that can actually participate in scattering events have wave vectors whose tips lie within the small sphere of size $k_B T/\hbar c$ about the tip of the wave vector **k**.

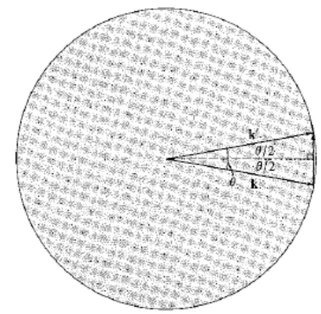
$$\hbar\omega(\mathbf{q}) << \varepsilon_F$$

Electron-Phonon Interaction

Since $\hbar\omega(\mathbf{q}) << \varepsilon_F$, the main effect of collision of an electron and a phonon is to change the direction of the electron wavevector **k**:

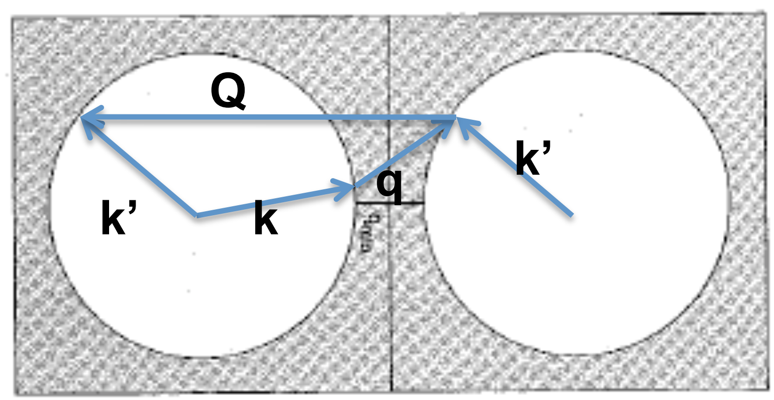
⌘ Normal process:

$$\mathbf{k}+\mathbf{q}=\mathbf{k}'$$



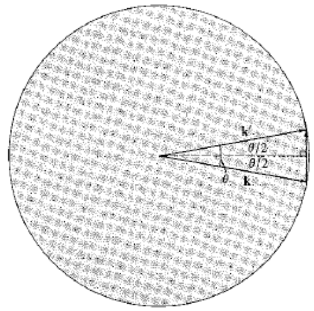
⌘ Umklapp process:

$$\mathbf{k}+\mathbf{q}=\mathbf{k}'+\mathbf{Q}$$



Electron-Phonon Interaction to Electrical Resistivity

$$\frac{1}{\tau_{el-ph}} = 2\pi k^2 \int_{FS} P(\theta)(1-\cos\theta)\sin\theta d\theta$$



$$\langle P(\theta)\rangle \propto \frac{k_B T}{\Theta_D^2}$$ -- scattering transition probability

$$\rho = \frac{AT^5}{M\Theta_D^6}\int_0^{\Theta_D/T} \frac{x^5 dx}{(e^x-1)(1-e^{-x})}$$

Bloch-Grüneisen formula

$$x = \frac{\hbar\omega_D}{k_B T}$$

Electron-Phonon Interaction to Electrical Resistivity at High T

$$x << 1$$

$$n(q) = \frac{1}{e\left(k_B T / \hbar\omega(q)\right) - 1} \approx \frac{k_B T}{\hbar\omega(q)}$$

$$\rho = \frac{AT}{4M\Theta_D^2}$$

$$\rho^{el-ph} \propto T \qquad T >> \Theta_D$$

Electron-Phonon Interaction to Electrical Resistivity at Low T

$$x >> 1$$

⬇

$$\rho = 124.4 \frac{AT^5}{M\Theta_D^6}$$

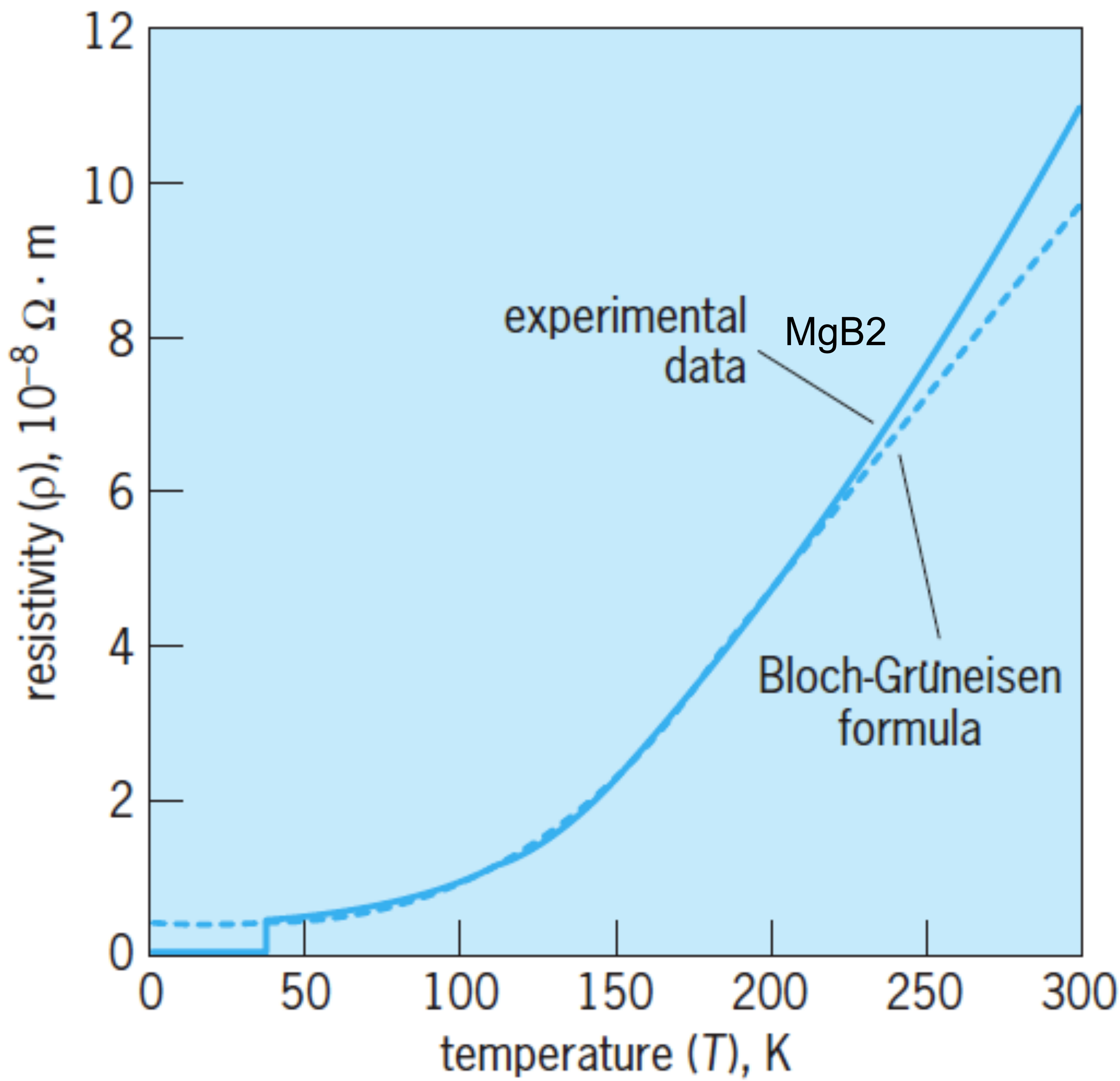
⬇

$$\rho^{el-ph} \propto T^5 \qquad T << \Theta_D$$

Total Electrical Resistivity in a Metal

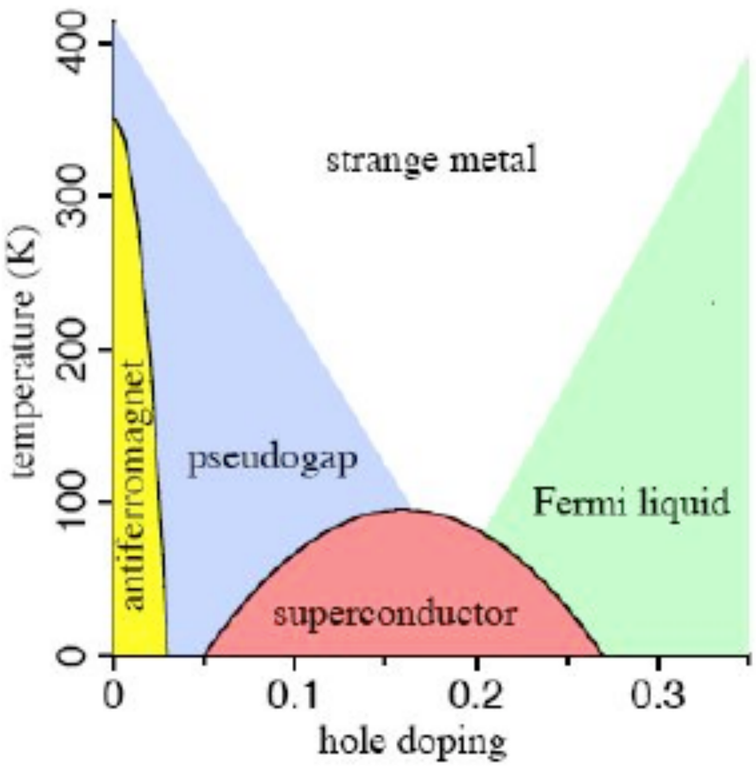
$$\rho = \rho_{el-el} + \rho_{el-ph}$$

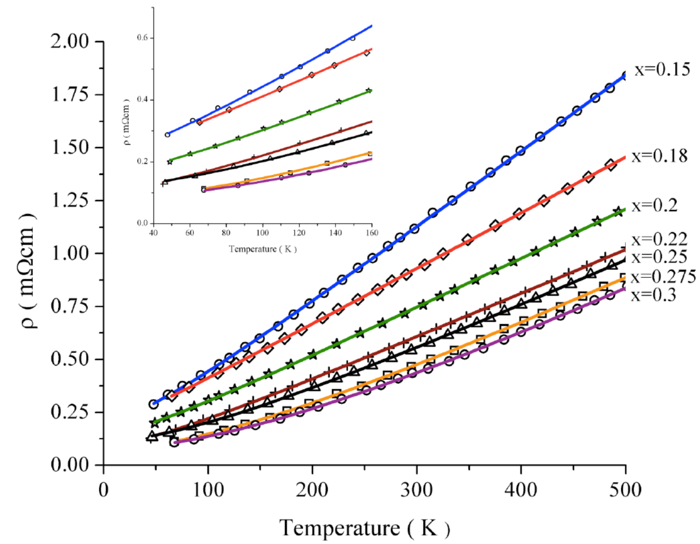
$$\rho = \begin{cases} \rho_{el-el} + BT \sim BT & T >> \Theta_D \\ \rho_{el-el} + BT^5 \sim \rho_{el-el} & T << \Theta_D \end{cases}$$

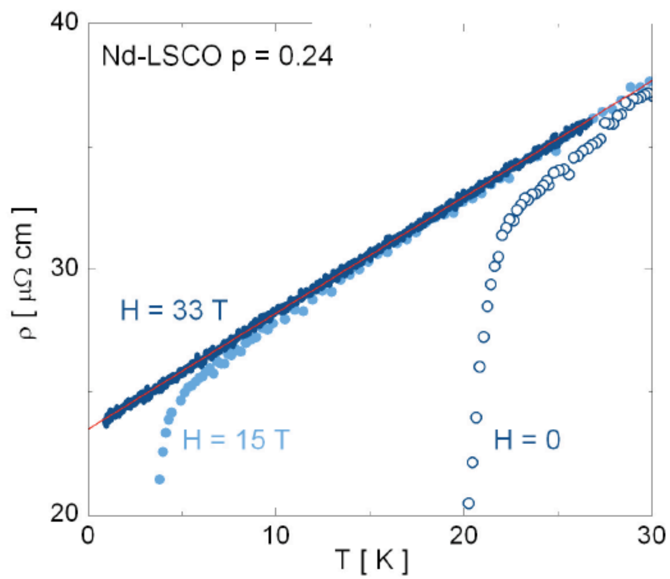


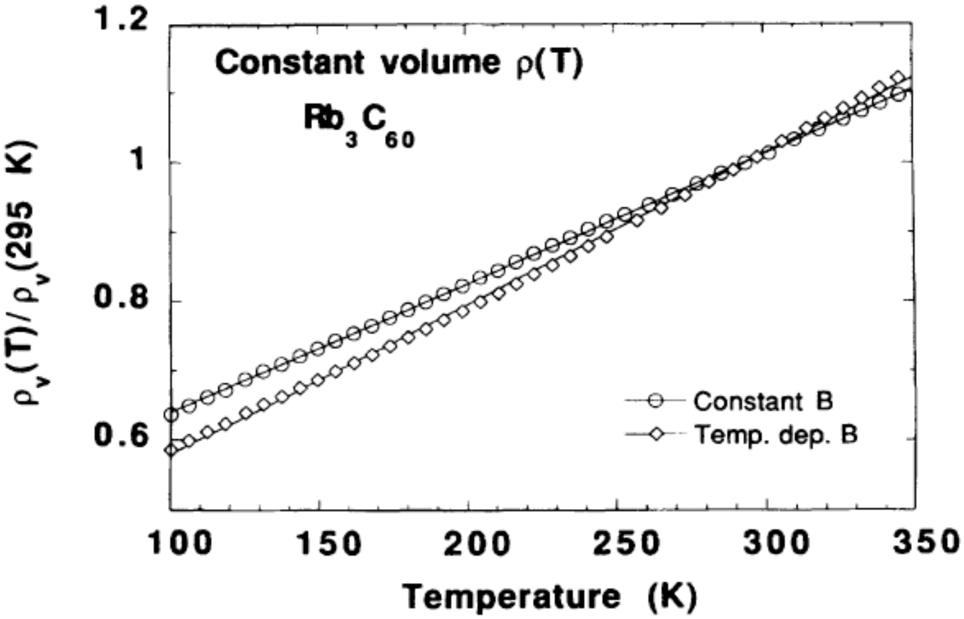
resistivity (ρ), 10⁻⁸ Ω · m
12
10
8
6
4
2
0
0
50
100
150
200
250
300
temperature (T), K
experimental data
MgB2
Bloch-Grüneisen formula

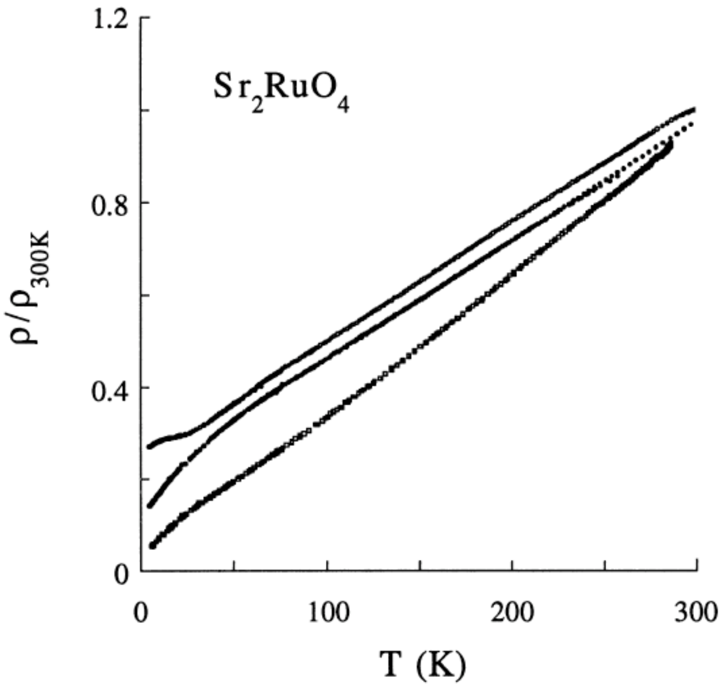
Strange Metals ?

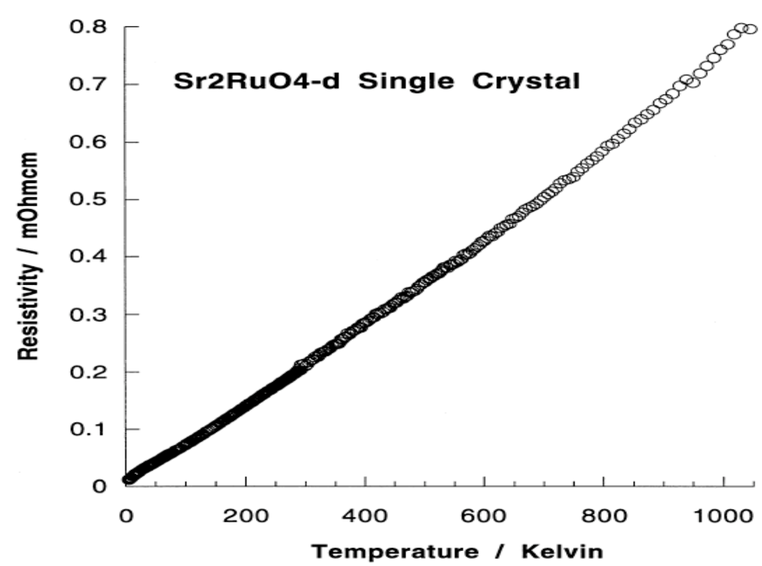







Constant volume ρ(T)
Rb3C60
ρv(T)/ρv(295 K)
1.2
1
0.8
0.6
100
150
200
250
300
350
Temperature (K)
Constant B
Temp. dep. B


Sr2RuO4
ρ/ρ300K
1.2
0.8
0.4
0
0
100
200
300
T (K)


Sr2RuO4-d Single Crystal
Resistivity / mOhmcm
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
0
200
400
600
800
1000
Temperature / Kelvin